A geometric perspective on duality symmetries in supergravity

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Based on 2311.12095 with

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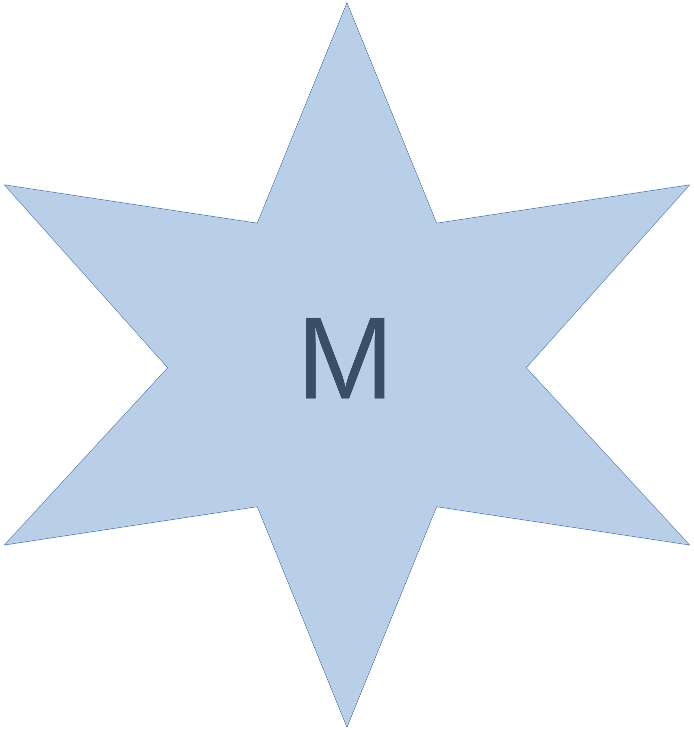
Introduction

Dualities in string & M-theory

11d SUGRA

$$S_{\text{eff}}^{(d=11)} = \frac{1}{2\kappa_{11}^2}\left[\int d^{11}x\,\sqrt{-G}\left(R - \frac{1}{2\cdot 4!}F_{M_1\ldots M_4}F^{M_1\ldots M_4}\right) - \frac{1}{6}\int A_3\wedge F_4\wedge F_4\right]. \qquad (16.131)$$



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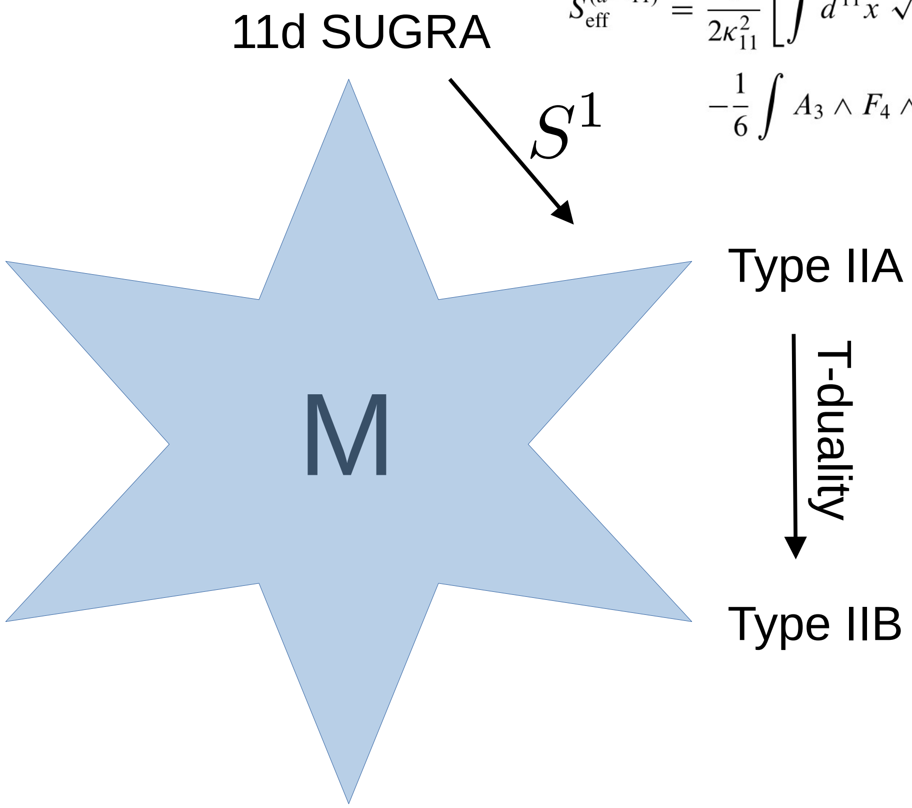

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$$S_{\text{IIB}} = \frac{1}{2\tilde{\kappa}_{10}^2}\int d^{10}x \sqrt{-G}\left[e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}|F_1|^2 - \frac{1}{2}|F_3|^2 - \frac{1}{4}|F_5|^2\right] - \frac{1}{4\tilde{\kappa}_{10}^2}\int C_4 \wedge H_3 \wedge F_3 \qquad (16.134b)$$

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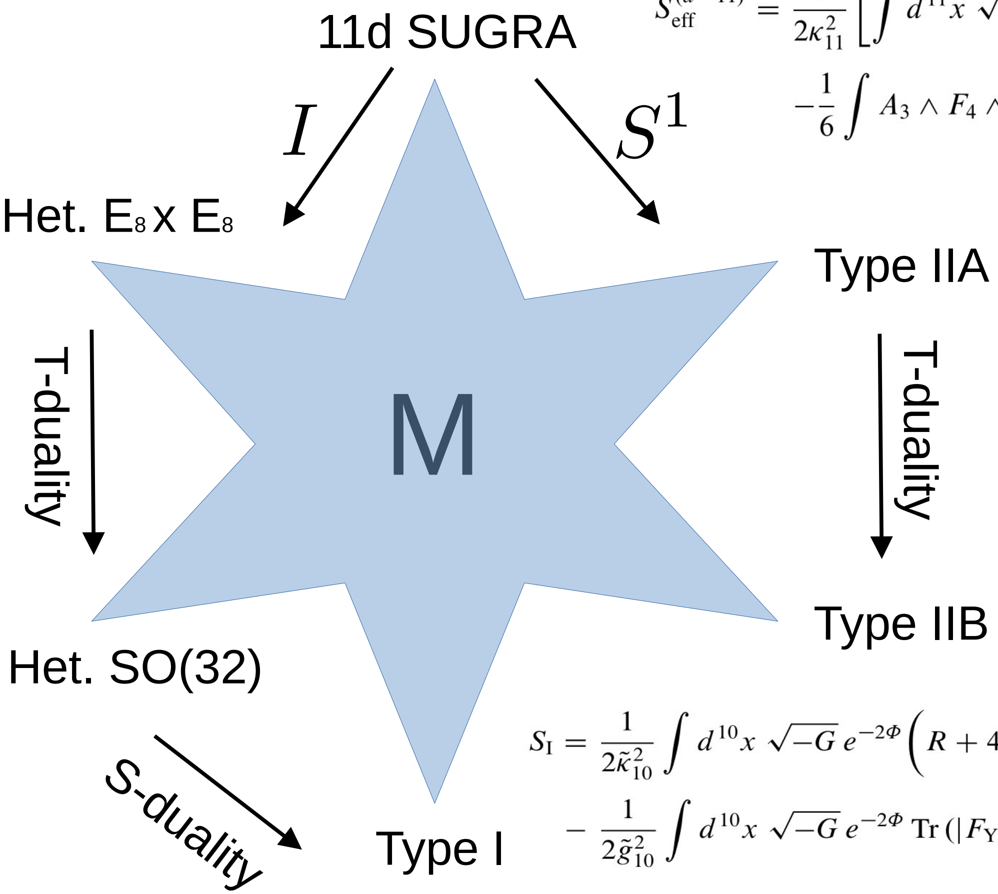

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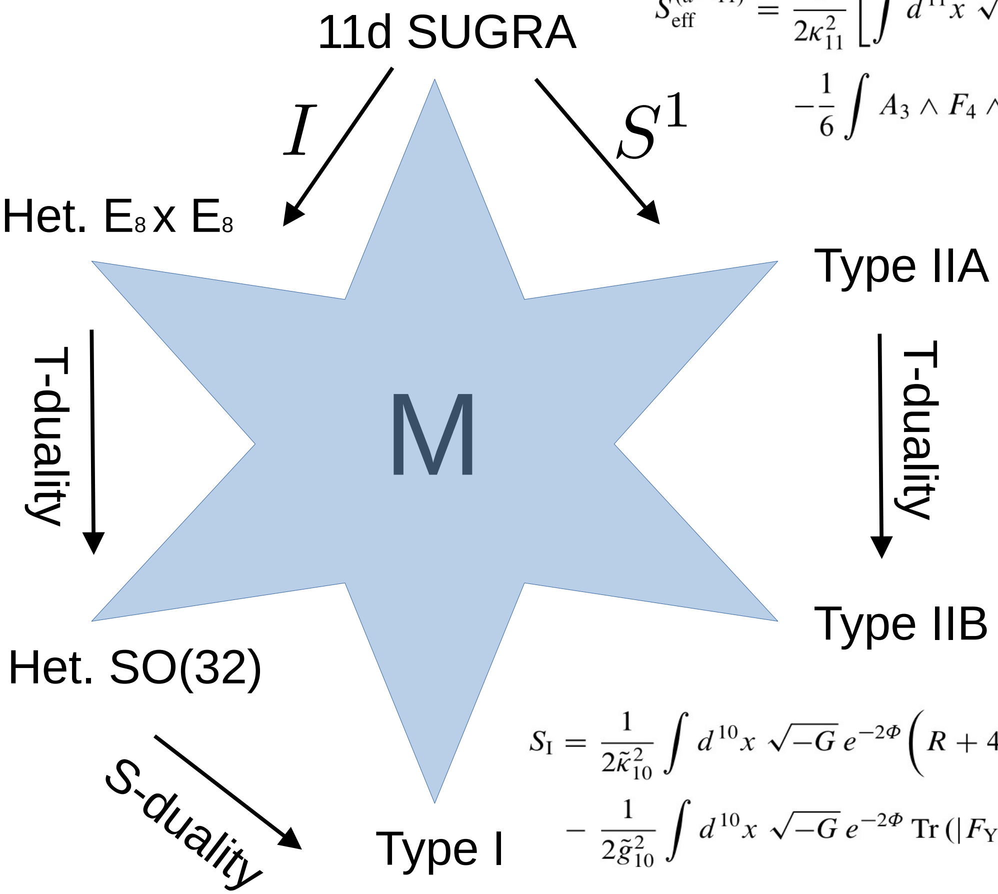
Dualities in string & M-theory



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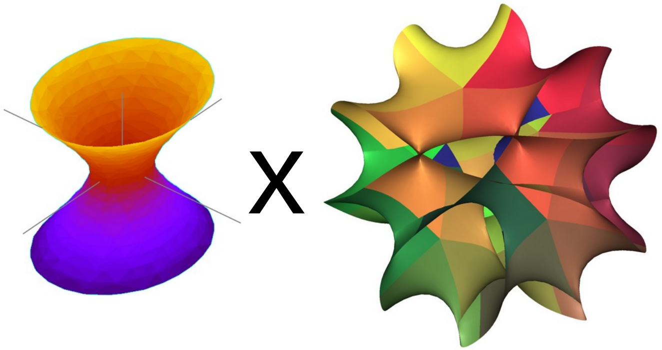
T- + S- = U-duality



Manifest U-duality: Fields

- split spacetime into 11 = n + d

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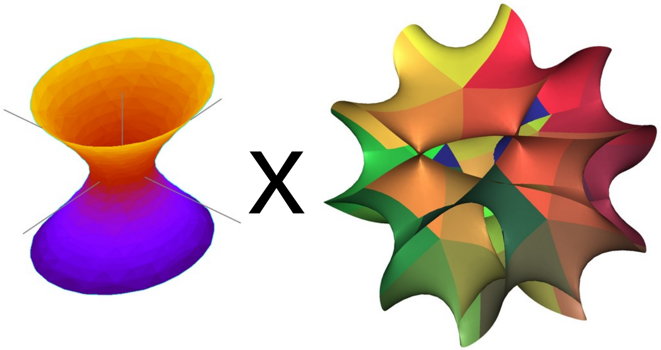

- split spacetime into 11 = n + d
- i.e. d=4 with U-duality group SL(5) and the multiplets

generalized metric

Metric	$g_{\mu\nu}$		
Scalars	$\mathcal{M}_{MN} \in \frac{\mathrm{SL}(5)}{\mathrm{SO}(5)}$	g_{ij} (10)	C_{ijk} (4)
One-forms	$\mathcal{A}_\mu \in \mathbf{10}$	$A_\mu{}^i$ (4)	$C_{\mu ij}$ (6)
Two-forms	$\mathcal{B}_{\mu\nu} \in \bar{\mathbf{5}}$	$C_{\mu\nu i}$ (4)	$C_{\mu\nu ijkl}$ (1)
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Four-forms	$\mathcal{D}_{\mu\nu\rho\sigma} \in \overline{\mathbf{10}}$	$C_{\mu\nu\rho\sigma ij}$ (6)	$\tilde{A}_{\mu\nu\rho\sigma i}$ (4)

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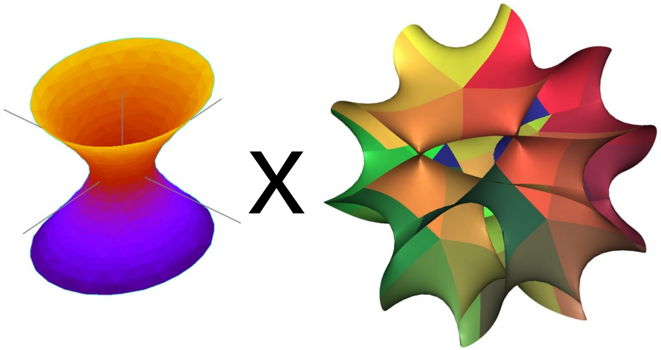

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generalized metric → $\mathcal{M}_{MN}$

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d	$E_{d(d)}$
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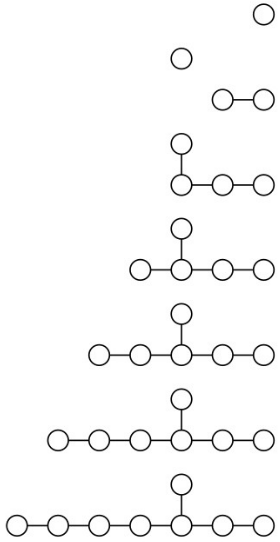
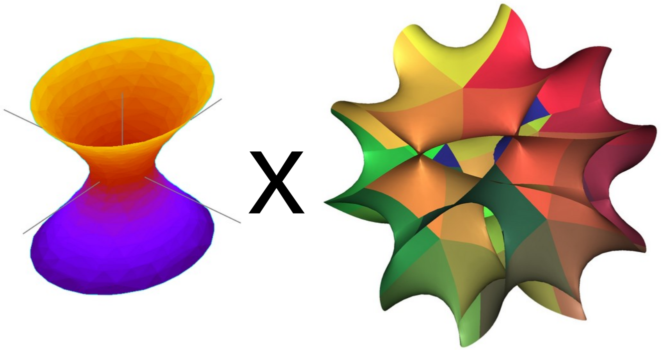
3	$\mathrm{SL}(3) \times \mathrm{SL}(2)$
4	$\mathrm{SL}(5)$
5	$\mathrm{SO}(5,5)$
6	$E_{6(6)}$
7	$E_{7(7)}$
8	$E_{8(8)}$





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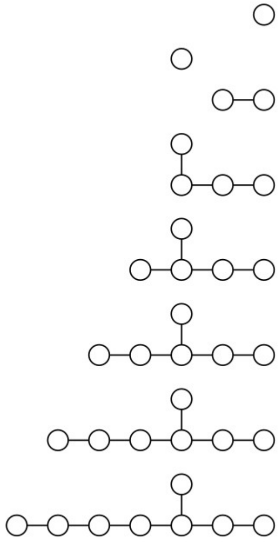
generalized metric

tensor hierarchy



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1) diffeomorphisms (gravity)
2) gauge tranformation

generalized Lorentz transformation

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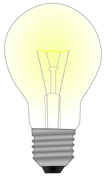

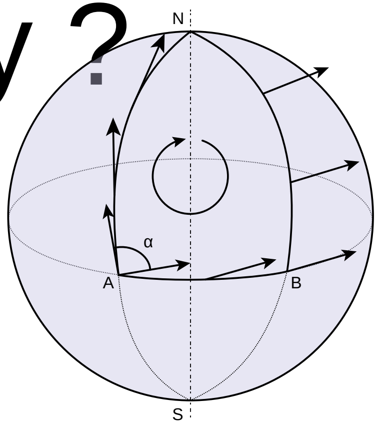

We have a Lie derivative… What about parallel transport?

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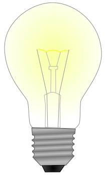

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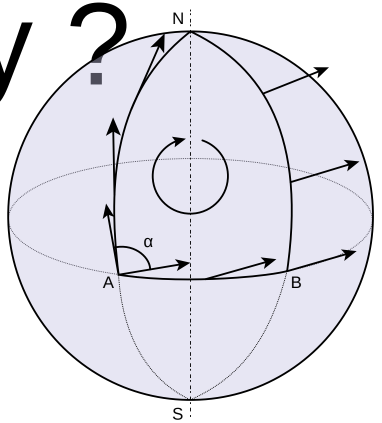

Covariant derivative:

$$\nabla_A E_B{}^M = E_A{}^N \partial_N E_B{}^M + \Omega_{AB}{}^C E_C{}^M - E_A{}^N \Gamma_{NL}{}^M E_B{}^L$$

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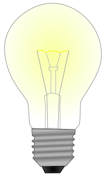

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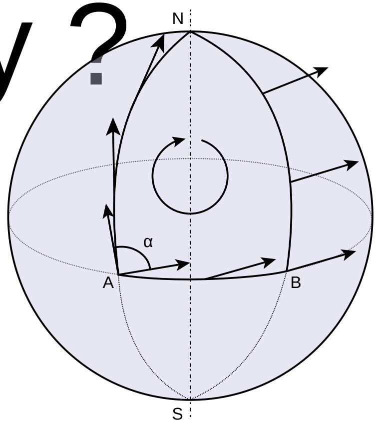

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Curvature and torsion???:

$$[\nabla_A, \nabla_B] V^C = R_{ABD}{}^C V^D + T_{AB}{}^D \nabla_D V^C$$

$R_{ABC}{}^D$ & $T_{AB}{}^C$

are not covariant

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A partial fix

- torsion can be quite easily defined:

$$T_{AB}{}^{C} E_C := \mathbb{L}^{\nabla}_{E_A} E_B - \mathbb{L}_{E_A} E_B$$

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$$S_{\text{eff}} = \cdots + \int \mathrm{d}^n x \mathrm{d}^d y \sqrt{g} R$$

gen. curvature scalar

$\mathrm{E}_{11(11)}$ gen. Einstein-Hilbert action

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Exceptional Generalized Geometry * 2007

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Exceptional Field Theory * 2012

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A hierarchy of curvatures

A better solution

The problem: find covariant curvature under gen. Lorentz tr. & gen. diffeomorphisms

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gen. structure group F tr.

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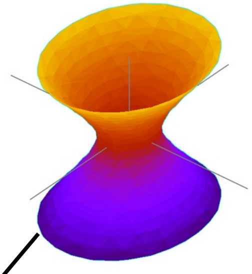
Observation: tensor hierarchy combines

In three words: symmetries for symmetries



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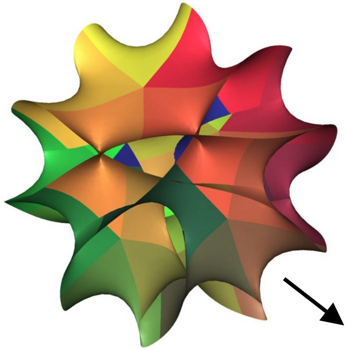

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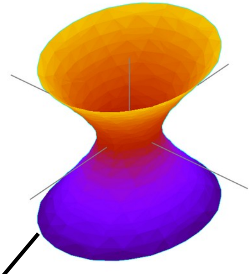

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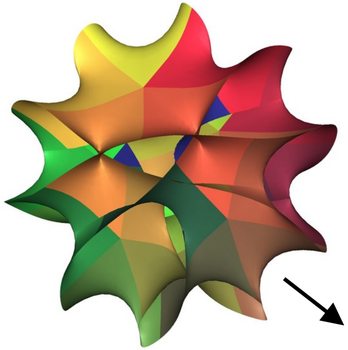

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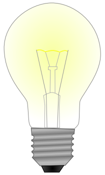

Re-purpose the tensor hierarchy to construct covariant curvature tensorS

Tensor hierarchy 101

- different approaches, best for our purpose is level decomposition

$$\mathrm{E}_{p(p)} \to \mathrm{E}_{d(d)} \times \mathrm{GL}(m)\,, \quad p = d + m$$

gen. diffs

contains gen. structure group $F \subset \mathrm{GL}(m)\,, m = \dim(F)$

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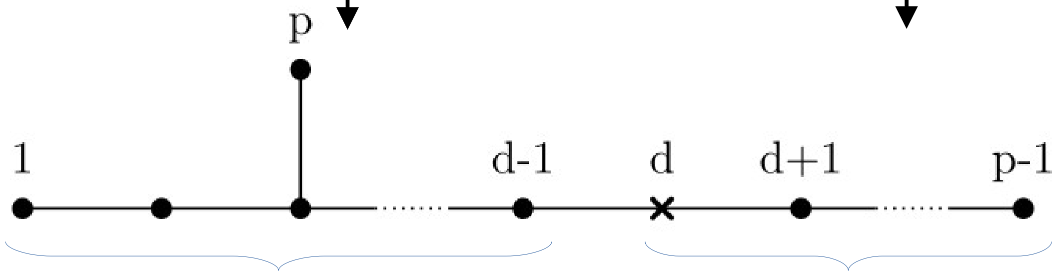
graded by $K_\alpha^\alpha = L$ (level)



$K_{\mathbf{a}}$ $\qquad$ K_α^β

$$[K_{\mathbf{a}}, K_{\mathbf{b}}] = f_{\mathbf{ab}}{}^{\mathbf{c}} K_{\mathbf{c}} \qquad [K_\alpha^\beta, K_\gamma^\delta] = \delta_\alpha^\delta K_\gamma^\beta - \delta_\gamma^\beta K_\alpha^\delta$$

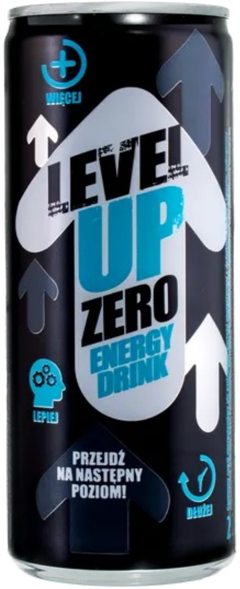
and this was just level 0 …

The next level

level 1 level -1

$$[\widetilde{R}^{\alpha}_{A}, R^{B}_{\beta}] = \delta^{B}_{A}\left(\beta\delta^{\beta}_{\alpha}L - K^{\alpha}_{\beta}\right) + \alpha\delta^{\alpha}_{\beta}(t^{\mathbf{a}})^{B}_{A}K_{\mathbf{a}}$$

R_1 representation of the duality group



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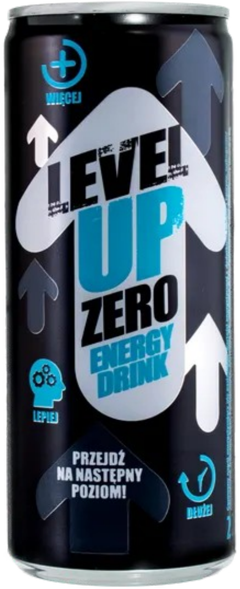

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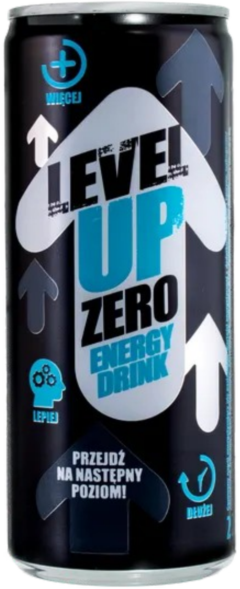

	O(d,d)	SL(5)	Spin(5,5)	$\mathrm{E}_{6(6)}$	$\mathrm{E}_{7(7)}$
α	2	3	4	6	12
β	0	1/5	1/4	1/3	1/2
R_1	**2d**	$\overline{\mathbf{10}}$	$\mathbf{16}_c$	**27**	**56**
R_2	**1**	**5**	**10**	$\overline{\mathbf{27}}$	**133**
R_3	–	**5**	$\mathbf{16}_s$	**78**	**912**

All levels beyond -1, 0, 1 are completely fixed by the Jacobi identity.

$E_{p(p)}$ generalized Lie derivative on the megaspace

- acts on the R_1 representation and its dual $\overline{R_1}$
- built from the highest/lowest weight state

$$|^{\alpha}\rangle \qquad\qquad \langle_{\alpha}|$$

$$\downarrow \qquad\qquad \downarrow$$

$$|^{A}\rangle = \frac{1}{n} R^{A}_{\alpha} |^{\alpha}\rangle \qquad\qquad \langle_{A}| = -\frac{1}{n} \langle_{\alpha}| \widetilde{R}^{\alpha}_{A}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\cdots \qquad\qquad \cdots$$

*) index-free version of $\mathrm{L}_U V^M = U^N \partial_N V^M - \alpha P_{(\mathrm{adj})}{}^M{}_{N,}{}^P{}_Q \partial_P U^Q V^N + \beta \partial_N U^N V^M$

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$$Z = C + \beta' \, 1 \otimes 1$$

duality group's split Casimir

$$\mathbb{L}_{\langle U|} \langle V| = \langle U|\partial_V\rangle\langle V| + \langle V|\langle U|Z|\partial_U\rangle$$

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Megaspace torsion

covarint under gen. diff

gen. Lorentz tr. & gen. diffeomorphisms

$\cup$

gen. structure group F tr.

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torsion

&

curvatures

on the phyiscal space

Exceptional Poláček-Siegel form

...or how to fix the frame I

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$$\widehat{E} = \widetilde{M} N \widetilde{V}$$

↙

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$X_\alpha = t_\alpha$ with $[t_\alpha, t_\beta] = X_{\alpha\beta}{}^\gamma t_\gamma$

Exceptional Poláček-Siegel form

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geometrizing the gen. structure group

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Exceptional Poláček-Siegel form

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$$\widehat{E} = \widetilde{M} N \widetilde{V}$$

$$N = \exp\left(\Omega_A^\alpha R_\alpha^A + \tfrac{1}{2}\rho^{\alpha\beta\overline{C}} R_{\alpha\beta\overline{C}+\dots}\right)$$

Exceptional Poláček-Siegel form

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nilpotent

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lower-triangular

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results in

lower-triangular

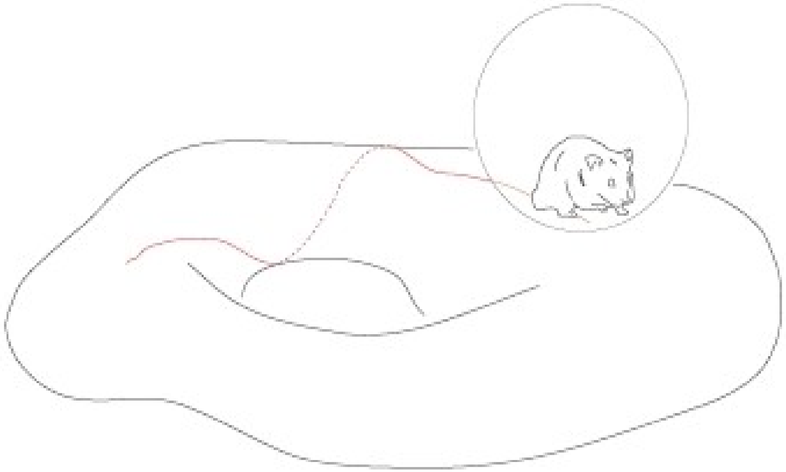
connections

$$X_A = X_A^{\mathbf{b}} K_{\mathbf{b}} + X_{AB}^{\beta} R_\beta^B + \tfrac{1}{2} X_A^{\beta_1\beta_2\overline{B}} R_{\beta_1\beta_2\overline{B}} + \dots$$

cov. cen. torsion Riemann tensor higher (derivative) curvature tensors

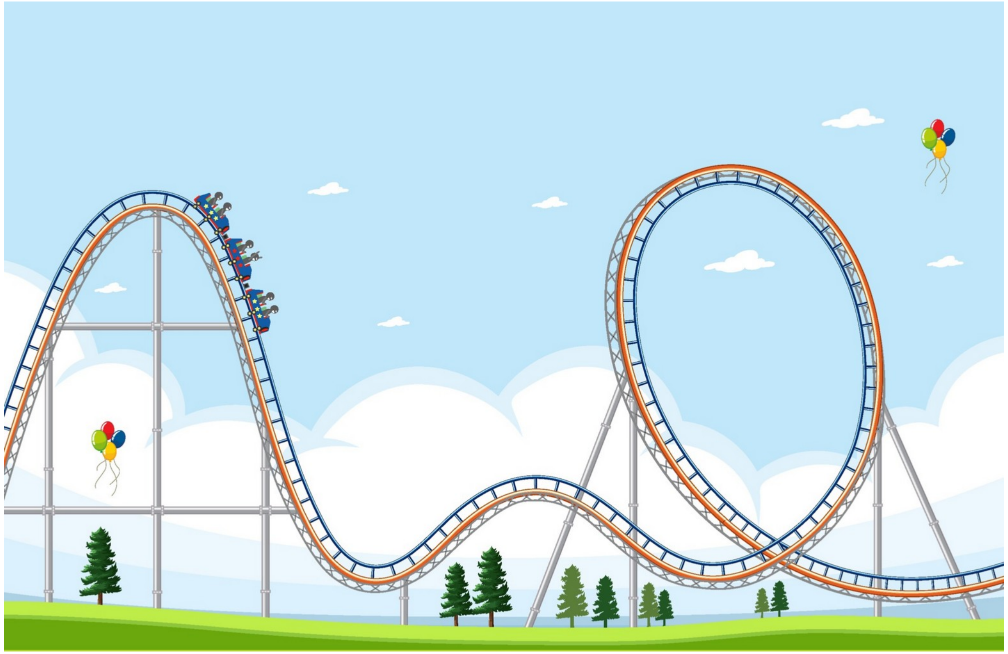
Possible Link?

Cartan geometry...



...unifies Torsion and curvature in a similar way.

Symplectic reduction...



...on the phase space of gauge theories has similar feature.

Applications

Dualities revisited

- Historical development of T-dualites

abelian $\subset$ non-abelian $\subset$ Poisson-Lie $\subset$ WZW-Poisson

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$\cap$ $\cap$

dressing coset $\subset$ **<u>generalized coset</u>**

generalized T-dualities

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$\cap$ (under Poisson-Lie) $\cap$ (under WZW-Poisson)

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generalized T-dualities

<u>Applications:</u>

- solution generating techniques
- consistent truncations
- integrable strings

Underlying structure

Felix Klein

Homogenious space: A space that looks everywhere the same as you move through it.

isometry → G/F ← isotropy

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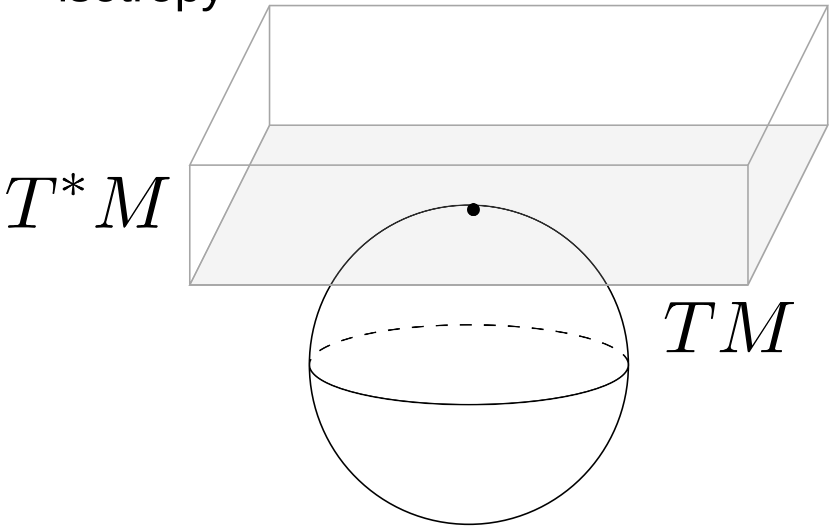
but in Generalized Geometry

with generalized Lie derivative: $\mathrm{L}_U V^M$

and section condition for closure



Generalized group manifold $\not{F}$

$$\mathbb{L}_{E_A} E_B{}^M = F_{AB}{}^C E_C{}^M$$

← gen. frame

↑ structure <u>constants</u>

Generalized group manifold $\not{F}$

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O(D,D) recipe to contruct gen. frame:

1) Lie algebra with generators T_A

$$[T_A, T_B] = F_{AB}{}^C T_C$$

2) with ad-invariant, O(D,D)-pairing

$$\langle T_A, T_B \rangle = \eta_{AB}$$

3) maximally isotropic subgroup

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O(D,D) recipe to contruct gen. frame:

embedding tensor of gSUGRA

&

<u>defining data of E-model</u>

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$$E_A{}^I$$

on coset

$$H \backslash G$$

Homogeneous space

<u>Theorem:</u> Let (M,g) be a connected and simply-connected complete Riemannian manifold. Then, the following statements are equivalent:

[Ambrose, Singer 1958]

1) The manifold M is Riemannian homogenous

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Riemann tensor

$S = \nabla^{\rm LC} - \nabla$

metric

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Riemann tensor → R; $S = \nabla^{\mathrm{LC}} - \nabla$ → S; metric → g

→ frame and <u>connection</u> required

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Generalized coset

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O(D,D) recipe to contruct gen. frame and spin connection:

1) gen. frame on H\G = mega-space

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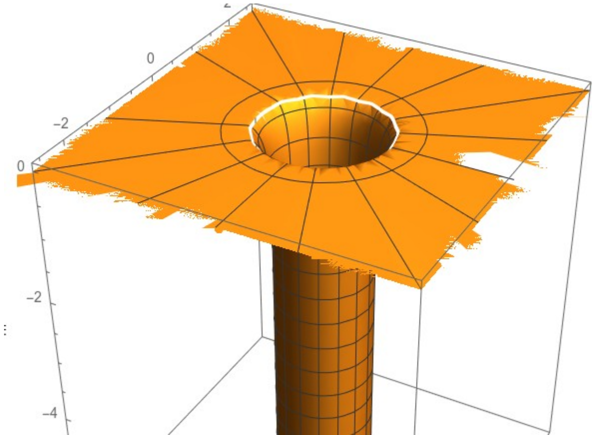

New

- *higher derivative* connections from *tensor hierarchy*
- *singularities* @ fixed points of F action

Summary and outlook

- finally covariant curvatures for execptional gen. geometry / field theory
- ulitmate goal is to use geometry, like in GR, to fix (as much as possible)

    1) target space low-energy effective action

    2) string and even membrane worldsheet theory

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integrable strings ⟷ generalized dualities ⟷ consistent truncations ?