

10. Spontaneous Symmetry Breaking

remember:

Symmetries in (Q)FT

global symmetries

local = gauge symmetries

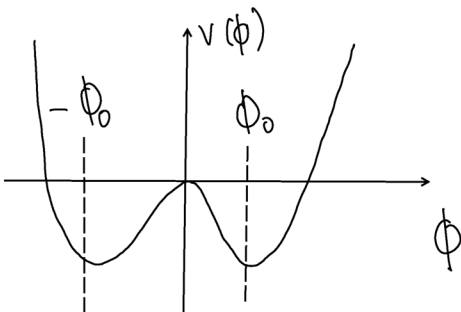
Today, we see why and how they can be broken.

10.1. Global Symmetries & Goldstone's Theorem

Example: $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!} \phi^4 = T - V$

introduce $m = i\mu$ in ϕ^4 -theory

Theory has a $\mathbb{Z}_2$ -symmetry $\phi \rightarrow -\phi$ and the potential



$$V = -\frac{1}{2} \mu^2 \phi^2 + \frac{\lambda}{4!} \phi^4$$

$$\phi_0 = \sqrt{\frac{6}{\lambda}} \mu \quad \text{with minimum } \pm \phi_0$$

called vacuum expectation value of ϕ

expanding $\phi(x) = \phi_0 + \sigma(x)$ results in

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 - \underbrace{\frac{1}{2} (2\mu^2) \sigma^2}_{\text{mass term for } m = \sqrt{2} \mu} - \underbrace{\sqrt{\frac{\lambda}{6}} \mu \sigma^3}_{\text{brakes } \mathbb{Z}_2\text{-Sym. } \phi \rightarrow -\phi} - \frac{\lambda}{4!} \sigma^4$$

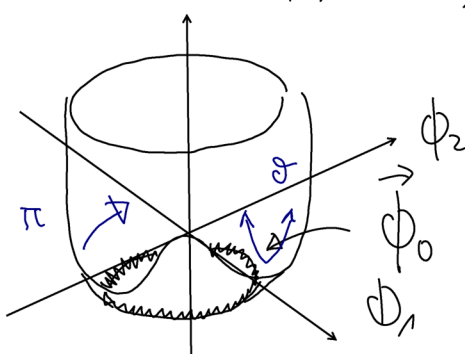
mass term for $m = \sqrt{2} \mu$ brakes $\mathbb{Z}_2$ -Sym. $\phi \rightarrow -\phi$

More interesting are continuous symmetries: $i=1, \dots, N$

like $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi^i)^2 + \frac{1}{2} \mu^2 (\phi^i)^2 - \frac{\lambda}{4} [(\phi^i)^2]^2 = T - V$

which has a $\phi^i \rightarrow R^i_j \phi^j$, $R^T R = \mathbb{1}$, $O(N)$ -symmetry

with $V(\phi^i) = -\frac{1}{2} \mu^2 (\phi^i)^2 + \frac{\lambda}{4} [(\phi^i)^2]^2$ and minima at



$$|\phi_0^i|^2 = \frac{\mu^2}{\lambda}$$

$\hat{=}$ vector $\vec{\phi}_0$ with fixed length

choose (hyper)-spherical coordinates such that

$$\phi(x) = \left(\underbrace{\frac{\mu}{\sqrt{\lambda}} + \mathcal{O}(x)}_{\text{radius}}, \underbrace{\pi^k(x)}_{\text{angles}} \right) \quad \text{and we hit the minima } \vec{\phi}_0 \text{ for } \mathcal{O}(x) = 0$$

results in:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \pi^k)^2 + \frac{1}{2} (\partial_\mu \mathcal{O})^2 - \frac{1}{2} (\lambda \mu^2) \mathcal{O}^2 - \sqrt{\lambda} \mu \mathcal{O}^3 - \sqrt{\lambda} \mu (\pi^k)^2 \mathcal{O} - \frac{\lambda}{4} \mathcal{O}^4 - \frac{\lambda}{2} (\pi^k)^2 \mathcal{O}^2 - \frac{\lambda}{4} [(\pi^k)^2]^2$$

- $\mathcal{O}$ is massive ($m_0 = \sqrt{\lambda} \mu$), like before
- π^k , $k=1, \dots, N-1$, massless fields with $O(N-1)$ -sym.

The observation from this example generalises:

For every spontaneously broken continuous symmetry, the theory must contain a massless particle

= Goldstone's theorem

check: $\dim(O(N)) = \frac{N(N-1)}{2}$, $\dim(O(N-1)) = \frac{(N-1)(N-2)}{2}$

broken symmetry generators = $\dim(O(N)) - \dim(O(N-1)) = N-1 = \# \pi^i_s$

Works not only classically, but to all orders in perturbation theory

10.2. Gauge Symmetries & The Higgs Mechanism

Simplest example: Abelian gauge theory with

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi), \quad \text{remember } D_\mu = \partial_\mu + ie A_\mu,$$

$$\text{and we find the local symmetry } \phi(x) \rightarrow e^{i\alpha(x)} \phi(x) \text{ and } A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x)$$

motivated by the examples from the last section we choose:

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2} (\phi^* \phi)^2 \text{ with minima at}$$

$$\langle \phi \rangle = \phi_0 = \left(\frac{\mu^2}{\lambda} \right)^{1/2}$$

again we expand around the minimum: $\phi(x) = \phi_0 + \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x))$
 with $V(\phi) = -\frac{1}{2\lambda} \mu^4 + \frac{1}{2} 2\mu^2 \phi_1^2 + \mathcal{O}(\phi_i^3)$

→ ϕ_1 acquires mass $m = \sqrt{2}\mu$
 ϕ_2 is the Goldstone boson, but something new happens for

kinetic term: $|\mathcal{D}_\mu \phi|^2 = \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 + \sqrt{2} e \phi_0 A_\mu \partial^\mu \phi_2 +$
 $\underbrace{e^2 \phi_0^2 A_\mu A^\mu}_{\text{new mass term for the photon}} + \dots$

new mass term for the photon $= \frac{1}{2} m_A^2 A_\mu A^\mu$
 with $m_A = \sqrt{2} e \phi_0$

Note: In contrast to the examples before, we can remove the Goldstone boson ϕ_2 by gauge fixing (unitary gauge).
 On the other hand the massive U(1)-gauge field allows a longitudinal mode (in contrast to the massless one ↗ section 3.5).

 The Goldstone boson gets "eaten up" by this mode.

Non-abelian generalisation:

gauge transformation: $\phi_i \rightarrow [\delta_i^j + i \underbrace{\mathcal{L}^a(t_a)}_{\text{here real fields}}]_{\substack{\uparrow \\ \text{purely imag.}}} \phi_j$

with covariant derivative: $\mathcal{D}_\mu \phi = (\partial_\mu - i g A_\mu^a t_a) \phi$ $T_a = i t_a$

kinetic term: $\frac{1}{2} (\mathcal{D}_\mu \phi_i)^2 = \frac{1}{2} (\partial_\mu \phi_i)^2 + g A_\mu^a (\partial^\mu \phi^i (T_a)_i^j \phi_j)$
 $+ \frac{1}{2} g^2 A_\mu^a A^{b\mu} (T^a \phi)^i (T^b \phi)_i$

expanding around VEV (vacuum expectation value) $\langle \phi_i \rangle = (\phi_0)_i$

→ new mass term $\Delta \mathcal{L} = \frac{1}{2} m_{ab}^2 A_\mu^a A^{b\mu}$ with
 $m_{ab}^2 = g^2 (T^a \phi_0)^i (T^b \phi_0)_i$

The matrix m_{ab}^2 is positive semidefinite $\rightarrow$ diagonalisable
with eigenvalues $\lambda_a \geq 0$

If $\lambda_a = 0$, $(T_a)_i^j (\phi)_j = 0$ unbroken symmetry

$\rightarrow$ no Goldstone boson & mass term for gauge boson

SU(2) example: $t_a = \frac{\sigma_a}{2} \leftarrow$ Pauli matrices

$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ results in the kinetic term

$$|D_\mu \phi|^2 = \frac{1}{2} g^2 \begin{pmatrix} 0 & v \end{pmatrix} t_a t_b \begin{pmatrix} 0 \\ v \end{pmatrix} A_\mu^a A_\nu^b + \dots$$

$$\leadsto \Delta \mathcal{L} = \frac{g^2 v^2}{8} A_\mu^a A_a^\mu \quad \leadsto \text{all three gauge bosons acquire the same mass}$$

$$m_A = \frac{g v}{2}$$