Quantum Field Theory, Summer 2023

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9. Renormalisation I (21 points)

To be discussed on Tuesday, 16th May, 2023 in the tutorial.
Please indicate your preferences until Thursday, 11/05/2023, 21:00:00 on the website.

We want to better understand how UV divergences arise in loop amplitudes. In the lecture, we used the superficial degree of divergence to find all "primitively" divergent Feynman diagrams in QED. In this exercise, we apply the same idea to another important model, the Yukawa theory.

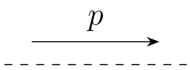
Exercise 9.1: Yukawa theory

We consider the following Lagrangian density,

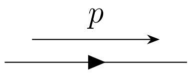
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_S^2\phi^2 + \overline{\psi}(i\partial\!\!\!/ - m_F)\psi - ig\overline{\psi}\gamma^5\psi\phi\,, \tag{1}$$

which couples a massive, real pseudoscalar field ϕ and a massive Dirac field ψ. A pseudoscalar behaves like a scalar, except that it changes sign under parity inversion. Therefore we have to add the γ^5 in the interaction term. Otherwise, (1) would not be invariant under party. The reason why we use a pseudoscalar is that pions (we discussed them in exercise 3) are pseudo scalars. Therefore, we can interpret ϕ as pion field that interact with the spin 1/2 hardron field ψ (capturing protons or neutrons). This way Yukawa theory describes the nuclear force between nucleons mediated by pions. Hideki Yukawa obtained the Nobel prize in 1949 "for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces."

a) (3 points) Determine the Feynman rules of the theory. *Hint: You should find the propagators*

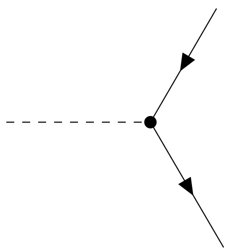


$$= \frac{i}{p^2 - m_S^2 + i\epsilon}\,,$$



$$= \frac{i(p\!\!\!/ + m)}{p^2 - m_F^2 + i\epsilon}\,,$$

and the interaction vertex



$$= -g\gamma^5\,.$$

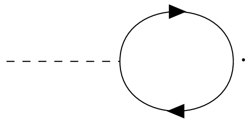
b) (3 points) Derive the formula for the superficial divergence for the Yukawa theory, like we did in the lecture for QED but for a variable dimension d of spacetime. *Hint: You should at then end obtain*

$$D = d + \frac{d-4}{2}V - \frac{d-1}{2}E_F - \frac{d-2}{2}E_S\,, \tag{2}$$

where V is the number of vertices, E_F the number of external fermionic legs and E_S the number of external scalar legs.

c) (3 points) Write down candidates for "primitively" divergent diagrams, like we did in the lecture for QED. *Hint: In total you should obtain 7 different diagrams.*

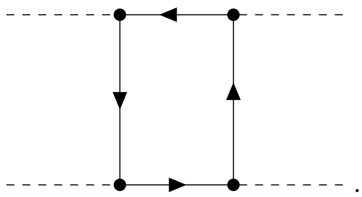
d) (3 points) We will now step by step analyse these diagrams. The vacuum diagram can be safely ignored because it drops out from all physical amplitudes. Consider now diagrams with one external, scalar leg. Take the simplest of these diagram, the one loop tad pole



Compute this diagram and argument why it is zero. *Hint: Show that* $\mathrm{Tr}\,\gamma^5 = \mathrm{Tr}\,\gamma^\mu\gamma^5 = 0$.

e) (3 points) This is not a coincidence and happens for higher loop diagrams, too. The Lagrangian (1) is invariant under parity (prove this statement and show how parity acts on the different fields). Assume that this symmetry is not broken after quantisation. Argument why therefore all correlation functions with an odd number of scalar legs (and no fermionic legs) have to vanish.

f) (3 points) There now four divergent diagrams left. For three of them, the divergences can be adsorbed by renormalising the parameters in the Lagrangian (1). However, this is not possible for the four scalar diagram. Let us therefore check if it is divergent. To this end, compute the one-loop Feynman diagram (with amputated legs)



In the limit $k^\mu \to \infty$, where k^μ denotes the four-momentum you integrate over, we can neglect all external momenta and masses in the numerator. We keep m_F in the numerator because there is still the chance that k^2 is small although individual components of k_μ are large. Show that we still get a logarithmic divergence for this diagram, as we also expect from its superficial divergence $D = 0$.

g) (3 points) To overcome this problem, we have to add the additional $\lambda/4!\phi^4$ to the Lagrangian (1). Compute the resulting new Feynman rule and show that still our analysis in c) stays valid.