Classical Field Theory, Winter 2025/26

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8. Pseudo - Euclidean space (18 points)

To be discussed on Wednesday, 10th December, 2025 in the tutorial.
Please indicate your preferences until Friday, 05/12/2025, 21:00:00 on the website.

Exercise 8.1: 2D Lorentz group

Consider a two-dimensional pseudo-Euclidian space.

a) (4 points) Find a matrix realization of the group of invariance of the metric i.e. prove that the Lorentz group in two dimensions can be written as:

$\mathcal{L}_+^\uparrow = \begin{pmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{pmatrix}$	$\mathcal{L}_+^\downarrow = \begin{pmatrix} -\cosh\phi & -\sinh\phi \\ -\sinh\phi & -\cosh\phi \end{pmatrix}$
$\mathcal{L}_-^\uparrow = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ \sinh\phi & -\cosh\phi \end{pmatrix}$	$\mathcal{L}_-^\downarrow = \begin{pmatrix} -\cosh\phi & \sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$

b) (3 points) Describe components of connectivity of this group.

c) (4 points) Which of these components are subgroups?
Hint: It is sufficient to show that the product of two elements of the same component is still of the same component. Use also the properties:

$$
\begin{aligned}
\sinh(x+y) &= \sinh x\cosh y + \cosh x\sinh y,\\
\cosh(x+y) &= \cosh x\cosh y + \sinh x\sinh y,\\
\sinh(x-y) &= \sinh x\cosh y - \cosh x\sinh y,\\
\cosh(x-y) &= \cosh x\cosh y - \sinh x\sinh y.
\end{aligned}
$$

d) (4 points) Find all subgroups of the $d = 2$ Lorentz group.
Hint: Some compositions (unions) of the components might also be subgroups.

e) (3 points) Describe geometrically sets of (Lorentz transformed) vectors of the same length.
Hint: Picturing them in a $x - t$ plane might be helpful.