Classical Field Theory, Winter 2023/24

Lecture: Prof. Andrzej Frydryszak, andrzej.frydryszak@uwr.edu.pl
Tutorial: Dr. Falk Hassler, falk.hassler@uwr.edu.pl
Assistant: M.Sc. Achilles Gitsis, achilleas.gitsis@uwr.edu.pl

9. Pseudo - Euclidean space (18 points)

To be discussed on Wednesday, 13th December, 2023 in the tutorial.
Please indicate your preferences until Friday, 08/12/2023, 21:00:00 on the website.

Exercise 9.1: 2D Lorentz group

Consider a two-dimensional pseudo-Euclidian space.

a) (4 points) Find a matrix realization of the group of invariance of the metric i.e. prove that the Lorentz group in two dimensions can be written as:

$\mathcal{L}_+^\uparrow = \begin{pmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{pmatrix}$	$\mathcal{L}_+^\downarrow = \begin{pmatrix} -\cosh\phi & -\sinh\phi \\ -\sinh\phi & -\cosh\phi \end{pmatrix}$
$\mathcal{L}_-^\uparrow = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ \sinh\phi & -\cosh\phi \end{pmatrix}$	$\mathcal{L}_-^\downarrow = \begin{pmatrix} -\cosh\phi & \sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$

b) (3 points) Describe components of connectivity of this group.

c) (4 points) Which of these components are subgroups?
Hint: It is sufficient to show that the product of two elements of the same component is still of the same component. Use also the properties:

$$\begin{aligned}
\sinh(x+y) &= \sinh x \cosh y + \cosh x \sinh y, \\
\cosh(x+y) &= \cosh x \cosh y + \sinh x \sinh y, \\
\sinh(x-y) &= \sinh x \cosh y - \cosh x \sinh y, \\
\cosh(x-y) &= \cosh x \cosh y - \sinh x \sinh y.
\end{aligned}$$

d) (4 points) Find all subgroups of the $d = 2$ Lorentz group.
Hint: Some compositions (unions) of the components might also be subgroups.

e) (3 points) Describe geometrically sets of (Lorentz transformed) vectors of the same length.
Hint: Picturing them in a $x - t$ plane might be helpful.