Classical Field Theory, Winter 2023/24

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8. Reference frames & relativistic motion (23 points)

To be discussed on Wednesday, 6th December, 2023 in the tutorial.
Please indicate your preferences until Friday, 01/12/2023, 21:00:00 on the website.

Exercise 8.1: Reference frames

A relativistic inertial reference frame $\mathcal{O}'$ moves with respect to another relativistic inertial reference frame $\mathcal{O}$ along the common x-axis with the velocity $\vec{u}$. A massive relativistic particle has velocity $\vec{v}$ and acceleration $\vec{a}$ with respect to the system $\mathcal{O}$.

Hint: Use the result of Ex. 7.2a. You can also set $c = 1$ for convenience.

a) (2 points) Prove that the transformed four-acceleration in $\mathcal{O}'$ is given by

$$w'^\mu = (\gamma_u w^0 - \beta_u \gamma_u w_x, -\beta_u \gamma_u w^0 + \gamma_u w_x, w_y, w_z), \qquad w^0 = \gamma_v^4 \frac{\vec{v} \cdot \vec{a}}{c},$$

where

$$\gamma_u = \frac{1}{\sqrt{1-\beta_u^2}}, \qquad \beta_u = \frac{u}{c}.$$

b) (4 points) Compute the x-component of the acceleration in $\mathcal{O}$, that is calculate a'_x.
Hint: Notice that the velocity v'_x that is perceived by an observer in $\mathcal{O}'$ is given by the velocity composition formula in Ex. 7.1a. You can also use the Lorentz transformation of the four-velocity to simplify your result.

c) (4 points) Compute the y-component of the acceleration in $\mathcal{O}$, that is calculate a'_y.
Hint: Use that

$$v'_y = \frac{1}{\gamma_u \left(1 - \frac{u v_x}{c^2}\right)} v_y.$$

Exercise 8.2: Relativistic motion

A relativistic particle moves in uniform circular motion

$$x^\mu = (t, r\cos\omega t, r\sin\omega t, 0), \qquad c = 1. \tag{1}$$

a) (3 points) Show that the transformed four-position according to an observer moving along y-axis with velocity $\vec{v}$ is

$$\begin{cases} t' = \frac{1}{\sqrt{1-u^2}} \frac{1}{\sqrt{1-v^2}} \tau - \frac{vr}{\sqrt{1-v^2}} \sin \frac{u\tau}{r\sqrt{1-u^2}} \\ x' = r\cos \frac{u\tau}{r\sqrt{1-u^2}} \\ y' = -v \frac{1}{\sqrt{1-u^2}} \frac{1}{\sqrt{1-v^2}} \tau + \frac{r}{\sqrt{1-v^2}} \sin \frac{u\tau}{r\sqrt{1-u^2}} \\ z' = 0 \end{cases}$$

where τ is the proper time and $u = \omega r$ is the tangential velocity.

b) (2 points) If this particle decays at rest with half-life time τ_0, what is the observed half-life time?

c) (4 points) Show that the magnitude of the proper acceleration 3-vector is given by

$$|\vec{\alpha}| = \frac{r\omega^2}{1 - r^2\omega^2}.$$

Hint: In a rest frame with a zero time component, the (square) magnitude of the proper acceleration 3-vector is the same as the (square) magnitude of the four-acceleration 3-vector. Use also the result of Ex. 7.2a and the properties of centripedal acceleration in uniform circular motion.

Exercise 8.3: Relativistic quantum mechanics

a) (4 points) Using arguments of relativistic mechanics, show that a free isolated electron cannot absorb or emit a photon.

Hint: Try to (dis)prove that by calculating the mass of the electron in terms of the photon's energy in the electron's rest frame. You can also set $c = \hbar = 1$ for convenience.