Selected Tools of Modern Theoretical Physics 2B, Summer 2025

Lecturer: Dr. Falk Hassler, falk.hassler@uwr.edu.pl
Tutorials: M.Sc. Alex Swash, alex.swash@uwr.edu.pl

7. Representations and Young tableaux (13 points)

To be discussed on Monday, 9th June, 2025 in the tutorial.
Please indicate your preferences until Wednesday, 04/06/2025, 21:00:00 on the website.

Exercise 7.1: Cartan matrix of a Lie algebra

Construct the Cartan matrices of the following Lie algebras:

a) (2 points) $\mathfrak{su}(3)$

b) (2 points) $\mathfrak{so}(4)$

c) (2 points) $\mathfrak{so}(5)$

d) (1 point) $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$.

Exercise 7.2: G2

Suppose a Lie algebra has the following simple roots,

$$\alpha_1 = (0, 1), \qquad \alpha_2 = (\sqrt{3}/2, -3/2). \tag{1}$$

a) (2 points) What is its Cartan matrix?

b) (1 point) What is the angle between the simple roots? Compare with the angles of the other rank 2 Lie algebras you know.

c) (2 bonus points) Reconstruct all other roots from these two simple roots (you may need to look up in a reference on how to do this, as this wasn't explained in detail during the lecture). What is the dimension of this Lie algebra?

Exercise 7.3: Properties of roots

a) (1 point) Prove that the number of positive roots is equal to the number of negative roots.

b) (2 points) Show that every root is an integral linear combination of *simple* roots, and that the coefficients are either all ≥ 0 or all ≤ 0.