



6. Four-velocity & four-acceleration (15 points)

To be discussed on Wednesday, 19th November, 2025 in the tutorial.

Please indicate your preferences until Friday, 14/11/2025, 21:00:00 on the website.

Exercise 6.1: Four-velocity

The four-velocity is defined as

$$u^\mu = \frac{dx^\mu}{d\tau} = \gamma(v)(c, \vec{v}), \quad \gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where $c^2 d\tau^2 = \eta_{\mu\nu} x^\mu x^\nu = c^2 dt^2 - d\vec{x}^2$

a) (3 points) Prove that the velocity composition formula is

$$v' = \frac{v - v'}{1 - \frac{vv'}{c^2}}. \quad (1)$$

Hint: Do so by applying a special Lorentz transformation to u^μ . Assume also 1D motion.

b) (2 points) Check whether (1) is associative.

Exercise 6.2: Four-acceleration

The four acceleration is defined as

$$w^\mu = \frac{du^\mu}{d\tau}.$$

a) (4 points) Express w^μ in terms of the acceleration $a^i = \frac{du^i}{dt}$ i.e. prove

$$w^\mu = \gamma^2(v) \left(\gamma^2(v) \frac{\vec{v} \cdot \vec{a}}{c}, \vec{a} + \gamma^2(v) \frac{\vec{v} \cdot \vec{a}}{c^2} \vec{v} \right).$$

Hint: Keep in mind that $dt = \gamma d\tau$.

b) (3 points) Calculate $w^\mu w_\mu$.

c) (3 points) Calculate $w^\mu u_\mu$.