Classical Field Theory, Winter 2023/24

Lecture: Prof. Andrzej Frydryszak, andrzej.frydryszak@uwr.edu.pl
Tutorial: Dr. Falk Hassler, falk.hassler@uwr.edu.pl
Assistant: M.Sc. Achilles Gitsis, achilleas.gitsis@uwr.edu.pl

7. Four-velocity & four-acceleration (15 points)

To be discussed on Wednesday, 29th November, 2023 in the tutorial.
Please indicate your preferences until Friday, 24/11/2023, 21:00:00 on the website.

Exercise 7.1: Four-velocity

The four-velocity is defined as

$$u^\mu = \frac{dx^\mu}{ds} = \gamma(v)(c, \vec{v}), \quad \gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where $ds^2 = \eta_{\mu\nu} x^\mu x^\nu = c^2 dt^2 - d\vec{x}^2$

a) (3 points) Prove that the velocity composition formula is

$$v' = \frac{v - \nu}{1 - \frac{v\nu}{c^2}}. \tag{1}$$

Hint: Do so by applying a special Lorentz transformation to u^μ. Assume also 1D motion.

b) (2 points) Check whether (1) is associative.

Exercise 7.2: Four-acceleration

The four acceleration is defined as

$$w^\mu = \frac{du^\mu}{ds}.$$

a) (4 points) Express w^μ in terms of the acceleration $a^i = \frac{du^i}{dt}$ i.e. prove

$$w^\mu = \gamma^2(v) \left(\gamma^2(v) \frac{\vec{v} \cdot \vec{a}}{c}, \vec{a} + \gamma^2(v) \frac{\vec{v} \cdot \vec{a}}{c^2} \vec{v} \right).$$

Hint: Keep in mind that $dt = \gamma ds$.

b) (3 points) Calculate $w^\mu w_\mu$.

c) (3 points) Calculate $w^\mu u_\mu$.