



5. Schwarz and triangle inequalities (11 points)

To be discussed on Wednesday, 12th November, 2025 in the tutorial.

Please indicate your preferences until Friday, 07/11/2025, 21:00:00 on the website.

Exercise 5.1: Schwarz inequality

Let $v, w \in M$ be causal vectors (that is not spacelike) and $\|v\| = \sqrt{\langle v, v \rangle}$.

a) (3 points) Show that

$$|\langle v, w \rangle| \geq \|v\| \|w\|. \quad (1)$$

Hint: Write

$$w = av + z, \quad z \in v^\perp$$

and use the result of Ex. 4.2b; calculating $\langle v, w \rangle^2$ might prove useful.

Exercise 5.2: Triangle inequality

Let $v, w \in M$ be vectors from the future cone.

a) (3 points) Show that

$$\|v + w\| \geq \|v\| + \|w\|. \quad (2)$$

The equality is achieved only for colinear vectors.

Hint: Use that $\langle v, w \rangle \geq 0$ along with the Schwarz inequality and the result of Ex. 4.1a.

b) (2 points) Is a similar relation valid for vectors from the past cone?

Exercise 5.3: Twin paradox

a) (3 points) Using (2) explain the twin paradox. In particular, explain why an argument (frequent in older literature) that, the acceleration (at the turning point of the trajectory of one of the twins) is responsible for the effect, should not be used.

Hint: Picture the vectors that each twin writes.