Quantum Field Theory, Summer 2024

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5. Interacting Scalar Theory (21 points)

To be discussed on Friday, 5th April, 2024 in the tutorial.
Please indicate your preferences until Sunday, 31/03/2024, 21:00:00 on the website.

Exercise 5.1: Propagator of ϕ^4 Theory

Consider the Klein-Gordon Lagrangian with an additional quartic interaction,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \,. \tag{1}$$

a) (3 points) Thinking in terms of the energy (looking at the interaction terms in the Hamiltonian might help here), why is a theory with a $g\phi^3$ interaction term not a viable option? Write down all relevant Feynman rules in momentum space. Demonstrate how we can use them to compute the correlation function

$$\langle 0|T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)|0\rangle$$

at the order $\mathcal{O}(\lambda^0)$ (without interactions), where we can just use Wick's theorem.

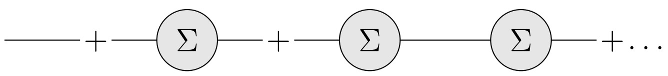
b) (3 points) To compute correlators of the interacting theory, we need to take into account the four-point vertex as well. Using it, draw all diagrams up to $\mathcal{O}(\lambda^2)$ with

1. no external legs,
2. connected diagrams with two external legs, and
3. all remaining diagrams with two external legs.

Specify the symmetry factor and the number of loop momenta for all diagrams in 1 and 2.

c) (3 points) The full propagator may be written as

$$D(p) = D_0(p).\sum_{n=0}^{\infty}\left(\Sigma(p).D_0(p)\right)^n = \tag{2}$$



where $D_0(p)$ denotes the propagator of the free theory. The function $\Sigma(p)$ is called the self-energy. It is the two-point one particle irreducible (1PI) function and it is described by the sum of 1PI diagrams. A 1PI diagram is one in which external legs are not separated if you cut a single line. Find the diagrams which contribute to $\Sigma(p)$ up to $\mathcal{O}(\lambda^2)$.

d) (3 points) Show that the full propagator (2) is consistent at $\mathcal{O}(\lambda^2)$ with the results from problem b). You will have to evaluate all symmetry factors to make sure the coefficients are correct.

e) (3 points) Show formally from (2) (by treating Σ as being an $\mathcal{O}(\lambda)$ object which is small in perturbation theory) that

$$D(p) = \frac{1}{D_0(p)^{-1} - \Sigma(p)} \,. \tag{3}$$

f) (3 points) The pole of the full propagator should be at the physical mass squared, $p^2 = m_{\text{phys}}^2$. Why? Hence, show that the physical mass is not the parameter m in the Lagrangian, but is given by the equation

$$m_{\text{phys}}^2 = m^2 - i\Sigma(p^2 = m_{\text{phys}}^2)\,.$$

g) (3 points) Let $\Sigma_1(p)$ be the lowest order contribution to Σ ($\mathcal{O}(\lambda^1)$). Analyse the value of $\Sigma_1(p)$. Do not calculate this in detail but argue that if we limit the size of the three- and four-momenta in the otherwise divergent integration to be $\mathcal{O}(\Lambda)$ or less, then $\Sigma_1 = c\Lambda^2$ and therefore is independent of the external momentum p. What does this tell us about the Lagrangian mass parameter m as $\Lambda \to \infty$?