Classical Field Theory, Winter 2023/24

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5. Minkowski space and the light cone (16 points)

To be discussed on Wednesday, 8th November, 2023 in the tutorial.
Please indicate your preferences until Friday, 03/11/2023, 21:00:00 on the website.

Exercise 5.1: Light cone

Let $(M, \langle\cdot,\cdot\rangle)$ be the Minkowski space; show that:

a) (4 points) For v and w lying in the same cone (either the future or the past one), i.e. $v, w \in \mathcal{J}^+$ (or $v, w \in \mathcal{J}^-$) such that $\langle v, v\rangle > 0$ and $\langle w, w\rangle \geq 0$ we have that

$$\langle v + w, v + w\rangle > 0.$$

Hint: Use the fact that both vectors lie on the same cone in combination with the Schwarz inequality

$$|\vec{v}||\vec{w}| \geq |\vec{v} \cdot \vec{w}|.$$

b) (3 points) Two lightlike vectors are orthogonal iff they are proportional.

Hint: The Schwarz inequality in the equality limit should be used at some point in the derivation.

Exercise 5.2: Subspaces of Minkowski space

Let $W \subset M$ be a subspace in M (the Minkowski space). Show the following statements:

Hint: For two non-degenerate subspaces of the Minkowski space $W = (W^\perp)^\perp$ and $W^\perp$, the former can be written as

$$M = W \oplus W^\perp$$

and it follows that

$$ind M = ind W + ind W^\perp, \tag{4}$$

where $ind V$ is the number of positive signs in the signature of the metric defined in V.

a) (3 points) W is timelike iff $W^\perp$ is spacelike.

Hint: Keep in mind that for timelike W, it is $ind W = 1$.

b) (3 points) W is spacelike iff $W^\perp$ is timelike.

Hint: Keep in mind that for spacelike W, it is $ind W = 0$.

c) (3 points) W is lightlike iff $W^\perp$ is lightlike.

Hint: Note that W is degenerate if it is lightlike.