Classical Field Theory, Winter 2023/24

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4. Initial value problem and Poisson brackets (16 points)

To be discussed on Wednesday, 25th October, 2023 in the tutorial.
Please indicate your preferences until Friday, 20/10/2023, 21:00:00 on the website.

Exercise 4.1: Initial value problem

We will now solve the initial value problem for the linear chain using normal coordinates, i.e. give an expression for $q_n(t)$ in terms of the initial conditions $q_n(0)$ and $\dot{q}_n(0)$.

a) (4 points) Express the coefficients b_k and $\bar{b}_k$ in terms of $q_n(t)$ and $\dot{q}_n(t)$.
Hint: You can easily do that by projecting these expansions onto the set of the basis functions, that is calculate

$$\sum_n e^{i\omega_k t}\bar{u}_n^k q_n(t)$$

and

$$\sum_n e^{i\omega_k t}\bar{u}_n^k \dot{q}_n(t)$$

b) (4 points) With

$$b_k = \frac{1}{2}\sum_n \bar{u}_n^k e^{i\omega_k t}\left(q_n(t) + \frac{i}{\omega_k}\dot{q}_n(t)\right)$$

and the abbreviation

$$G_{nn'}(t) = \sum_k e^{-i\omega_k t} u_n^k \bar{u}_{n'}^k = \frac{1}{N}\sum_k e^{i[ka(n-n')-\omega_k t]},$$

prove that the solution of the initial value problem reduces to

$$q_n(t) = \frac{1}{N}\sum_k \sum_n \left\{ q_{n'}(0)\cos\left[ka(n-n') - \omega_k t\right] - \frac{1}{\omega_k}\dot{q}_{n'}(0)\sin\left[ka(n-n') - \omega_k t\right]\right\}.$$

Exercise 4.2: Poisson brackets

Last but not least, we calculate the Poisson brackets, since we have everything we need. Remember the definition of the Poisson brackets

$$\{A, B\} := \sum_{i=1}^{N}\left(\frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i}\right),$$

where $A = A(q_i, p_i, t)$ and $B = B(q_i, p_i, t)$.

a) (4 points) Calculate $\{b_k, \bar{b}_{k'}\}$.

b) (4 points) Calculate $\{b_k, b_{k'}\}$ and $\{\bar{b}_k, \bar{b}_{k'}\}$ as well.