Gauge/Gravity Duality, Winter 2024/25

Lecturer: Dr. Falk Hassler, falk.hassler@uwr.edu.pl
Tutorials: M.Sc. Luca Scala, luca.scala@uwr.edu.pl

12. The AdS/CFT correspondence. (16 points)

To be discussed on Friday, 17th January, 2025 in the tutorial.
Please indicate your preferences until Monday, 13/01/2025, 21:00:00 on the website.

Exercise 12.1: D_p-brane metric.

The extremal black D_p-brane metric is given by

$$\mathrm{d}s^2 = H_p^{-1/2}\mathrm{d}x^2 + H_p^{1/2}\mathrm{d}y^2,$$

where $H_p(r) = 1 + \left(\frac{L_p}{r}\right)^{7-p}$, dx^2 is the $(p+1)$-dimensional Lorentz metric along the brane and dy^2 is the Euclidean metric in the $9-p$ directions. It is complemented by the dilaton field

$$e^\phi = g_s H_p^{(3-p)/4},$$

and the $(p+1)$-form gauge potential

$$C_{(p+1)} = \left(H_p^{-1} - 1\right)\mathrm{d}x^0 \wedge \cdots \wedge \mathrm{d}x^p\,. \tag{1}$$

a) (4 points) Given the type IIB action for a D3-brane

$$S = \frac{1}{16\pi G}\int d^{10}x\sqrt{g}\left(R - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{4}|F_5|^2\right),$$

find the equations of motion. In particular, you should find that for the 5-form field strength

$$\nabla^M F_{MNPQR} = 0 \tag{2}$$

holds.

b) (3 points) Show that using differential forms, like

$$F_{(5)} = \frac{1}{5!}F_{MNPQR}\mathrm{d}x^M \wedge \cdots \wedge \mathrm{d}x^R\,,$$

you can write (2) as

$$\star\,\mathrm{d}\,\star F_{(5)} = 0\,,$$

where $\star$ is the Hodge star operator already discussed in exercise sheet 3.1. A central property of type IIB supergravity is that $F_{(5)}$ has to be self-dual, saying that it is governed by

$$F_{(5)} = \star F_{(5)}\,.$$

Check that this property holds for $F_{(5)} = \mathrm{d}C_{(4)}$ with the 4-form gauge potential of a D3-brane given in (1). Clearly this implies the Bianchi identity $\mathrm{d}F_{(5)} = 0$. Why? What does this tell us about the field equation for the 5-form field strength (2)?

c) (4 points) Check that, the D3-brane background solve also the remaining two equations of motion

$$\nabla^M \nabla_M \phi = 0\,,$$
$$G_{MN} - \frac{1}{2} T^F_{MN} - \frac{1}{2} T^\phi_{MN} = 0\,,$$

where

$$T^F_{MN} = \frac{1}{4!} F_{MOPQR} F_N{}^{OPQR} - \frac{1}{2} g_{MN} F^2,$$
$$T^\phi_{MN} = \partial_M \phi \partial_N \phi - \frac{1}{2} g_{MN} (\partial\phi)^2,$$

$F^2 = \frac{1}{5!} F_{MNOPQ} F^{MNOPQ}$ and G_{MN} is the ordinary Einstein tensor.

d) (3 points) Check that for $p = 3$ the near-horizon limit of that metric gives the metric of an $AdS_5 \times S^5$ space, setting $L_3 = R$.

Exercise 12.2: The AdS/CFT correspondence. 2 points

String corrections to the gravity action come as g_s corrections to terms already present while α' corrections appear generally as $(\alpha' R)^n$, with R some particular contractions of Riemann tensors. To what correspond, then, α' and g_s corrections in Super Yang-Mills via AdS/CFT (in the $N \to \infty$, $\lambda = g_{YM}^2 N$ fixed and large limit)?

Bonus Exercise 12.3: AdS_5/CFT_4. 4 bonus points

Show that the number of degrees of freedom per site in the CFT_4 is proportional to the size of the AdS_5 boundary:

$$N^2 \propto \frac{L^3}{G_N^5},$$

where L is the AdS radius and G_N^5 the Newton constant in 5D, obtained by taking the 10D one $G_N^{10} = \frac{(2\pi)^7 \alpha'^4 g_s^2}{16\pi}$ and dividing by the volume of S^5. *Hint: This is something which goes beyond what was done already in the lecture. But that is what bonus problems are for. It should be still possible to solve it by look at section 6.3 in the book "Gauge/Gravity Duality" by Ammon and Erdmenger.*