Classical Field Theory, Winter 2025/26

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11. Electrodynamics (20 points)

To be discussed on Wednesday, 14th January, 2026 in the tutorial.
Please indicate your preferences until Friday, 09/01/2026, 21:00:00 on the website.

Exercise 11.1: Covariant electrodynamics

Let

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and explicit identifications of the electric and magnetic fields are given by

$$\begin{aligned} F^{0i} &= -E^i \\ F^{ij} &= -\epsilon^{ijk} B^k. \end{aligned}$$

a) (3 points) Prove that the equations of motion are given by

$$\partial_\mu F^{\mu\nu} = 0.$$

b) (3 points) Prove that the canonical energy-momentum tensor for the Lagrangian density defined above is given by

$$T^{\mu\nu} = \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - F^{\mu\lambda} F^{\nu}{}_{\lambda} - F^{\mu\lambda} \partial_\lambda A^\nu.$$

c) (2 points) Show that $T^{\mu\nu}$ is not symmetric.

d) (3 points) One can remedy this by considering a new tensor

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu}, \qquad K^{\lambda\mu\nu} = F^{\mu\lambda} A^\nu.$$

Find out that $\tilde{T}^{\mu\nu}$ is symmetric.

e) (3 points) Show that from $\tilde{T}^{\mu\nu}$ one can reproduce the standard expression for the energy density, that is

$$\mathcal{E} = \frac{1}{2}(E^2 + B^2).$$

Hint: Use $\epsilon_{ijk}\epsilon_{ijl} = 2\delta_{kl}$.

f) (3 points) Do the same as in the previous task but with the momentum density, i.e.

$$\vec{S} = \vec{E} \times \vec{B}.$$

g) (3 points) Calculate the trace of $\tilde{T}^{\mu\nu}$.