Quantum Field Theory, Summer 2024

Lecturer: Dr. Falk Hassler, falk.hassler@uwr.edu.pl
Tutorials: Prof. UWr Pok Man Lo, pokman.lo@uwr.edu.pl
Assistant: M.Sc. Alex Swash, alex.swash@uwr.edu.pl

10. Renormalisation II (18 points)

To be discussed on Friday, 24th May, 2024 in the tutorial.
Please indicate your preferences until Sunday, 19/05/2024, 21:00:00 on the website.

In the lecture, we discussed how to renormalise ϕ^4-theory, which is defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \,. \tag{1}$$

Although we covered all required steps, there was not enough time to look at all the technical details. Therefore, we will look here at all the details. Please take this as an opportunity to revisit all the techniques we learned so far. Even if you have not been assigned to a specific task, it is highly recommended that you try to solve the full problem set to see if you have any difficulties with the path integral, Feynman rules or regularisation.

Exercise 10.1: One-loop renormalisation of ϕ^4 theory

We follow Peskin and Schroeder chapter 10.2.

a) (3 points) First, we have to obtain the counterterms and their Feynman rules. To this end, we take the Lagrangian and replace the mass m and the coupling λ by the bare quantities, denoted by m_0 and λ_0, to obtain

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}\phi^4 \,.$$

Next, we substitute ϕ by the renormalised field

$$\phi = Z^{1/2}\phi_\mathrm{r}$$

and eliminate the bare parameters m_0 and λ_0 by defining

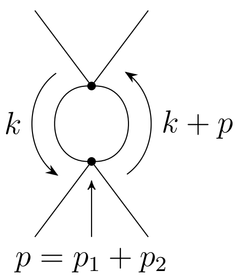
$$\delta_Z = Z - 1 \,, \qquad \delta_m = m_0^2 Z - m^2 \,, \quad \text{and} \quad \delta_\lambda = \lambda_0 Z^2 - \lambda \,.$$

Compute the resulting Lagrangian and derive its Feynman rules (the propagator for ϕ_r, its four-point vertex and the corresponding counterterms).

Next, we compute two fundamental one-loop diagrams:

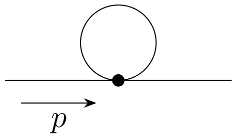
b) (3 points) Write down $V(p^2)$ for the diagram

$$(-i\lambda)^2 iV(p^2) \equiv$$



and compute the integral by introducing Feynman parameters, shifting the integration variable, rotating to Euclidean space and applying dimensional regularisation for $\epsilon = 4 - d$.

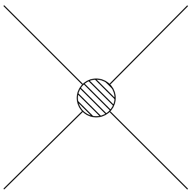
c) (3 points) Write down $M_1(p^2)$ for the diagram

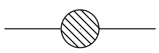
$$-iM_1(p^2) =$$  ,

rotate to Euclidean space and compute the integral by dimensional regularisation.

d) (3 points) Use the results from the two previous tasks to calculate the four-point interaction and the propagator, including all relevant counterterms, up to order λ.

e) (3 points) Impose the renormalisation conditions

 $$= -i\lambda \quad \text{at} \quad s = 4m^2\,, t = u = 0\,, \quad \text{and}$$

 $$= \frac{i}{p^2 - m^2} + (\text{terms regular at } p^2 = m^2)$$

to fix δ_λ, δ_Z and δ_m (here s, t and u are Mandelstam variables).

f) (3 points) Use the values for δ_λ, δ_Z and δ_m calculated in the previous task to finally obtain the renormalised propagator (depending on the momentum p) and four-point interaction (depending on the Mandelstam variables s, t and u) up to order λ.

g) (3 bonus points) There is a nice Mathematica package called FeynCalc. It is able to automate most of the tedious computations we have done here. More precisely, you might want to look at the example

`https://feyncalc.github.io/FeynCalcExamplesMD/Phi4/OneLoop/Renormalization` ,

which deals exactly with the questions posed in the previous tasks. Try to get this package running on your computer and compare the results with what you (or Peskin and Schroeder) got by hand.