



Supergeneralized geometry and dualities

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Based on work with

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String dualities

- Historical development of T-dualities

abelian \subset non-abelian \subset Poisson-Lie \subset WZW-Poisson

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dressing coset \subset **generalized coset**



generalized T-dualities

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generalized T-dualities

Applications:

- solution generating techniques
- consistent truncations
- *integrable strings*

Underlying structure



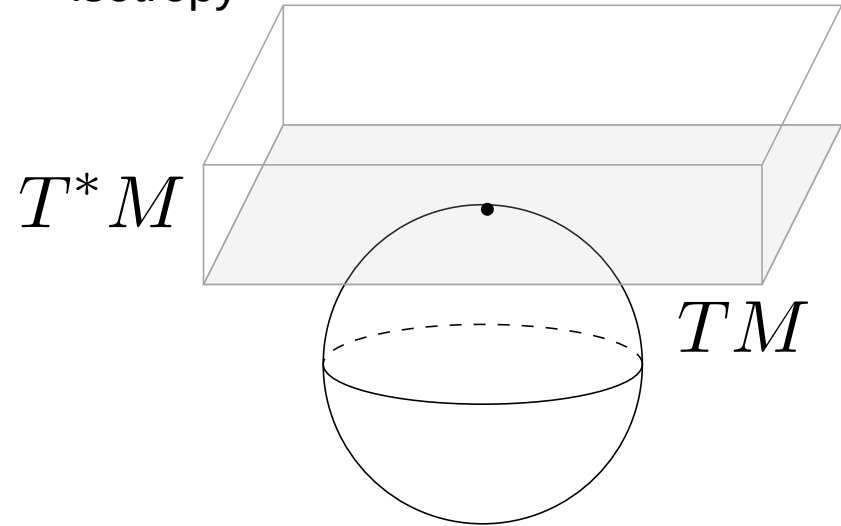
Felix Klein

Homogenous space:

A space that looks everywhere the same as you move through it.

$$\text{isometry} \longrightarrow G/F \longleftarrow \text{isotropy}$$

but in **Generalized Geometry**





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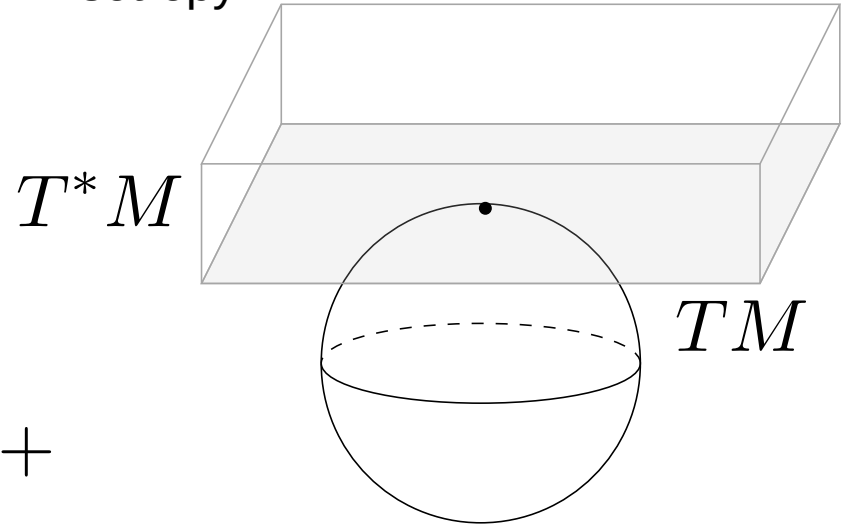
generalized Lie derivative:

$$\mathbb{L}_U V^M = U^N \partial_N V^M -$$

$$\alpha P_{(\text{adj})}{}^M{}_{N, P}{}^Q \partial_P U^Q V^N +$$

$$\beta \partial_N U^N V^M$$

+ section condition for closure



Generalized group manifold



$$\mathbb{L}_{E_A} E_B^M = F_{AB}^C E_C^M$$


← gen. frame

↑ structure constants

Generalized group manifold



$$\mathbb{L}_{E_A} E_B^M = F_{AB}^C E_C^M \leftarrow \text{gen. frame}$$

 structure constants

O(D,D) recipe to construct gen. frame:

1) Lie algebra with generators T_A

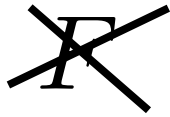
$$[T_A, T_B] = F_{AB}^C T_C$$

2) with ad-invariant, O(D,D)-pairing


$$\langle T_A, T_B \rangle = \eta_{AB}$$

3) maximally isotropic subgroup

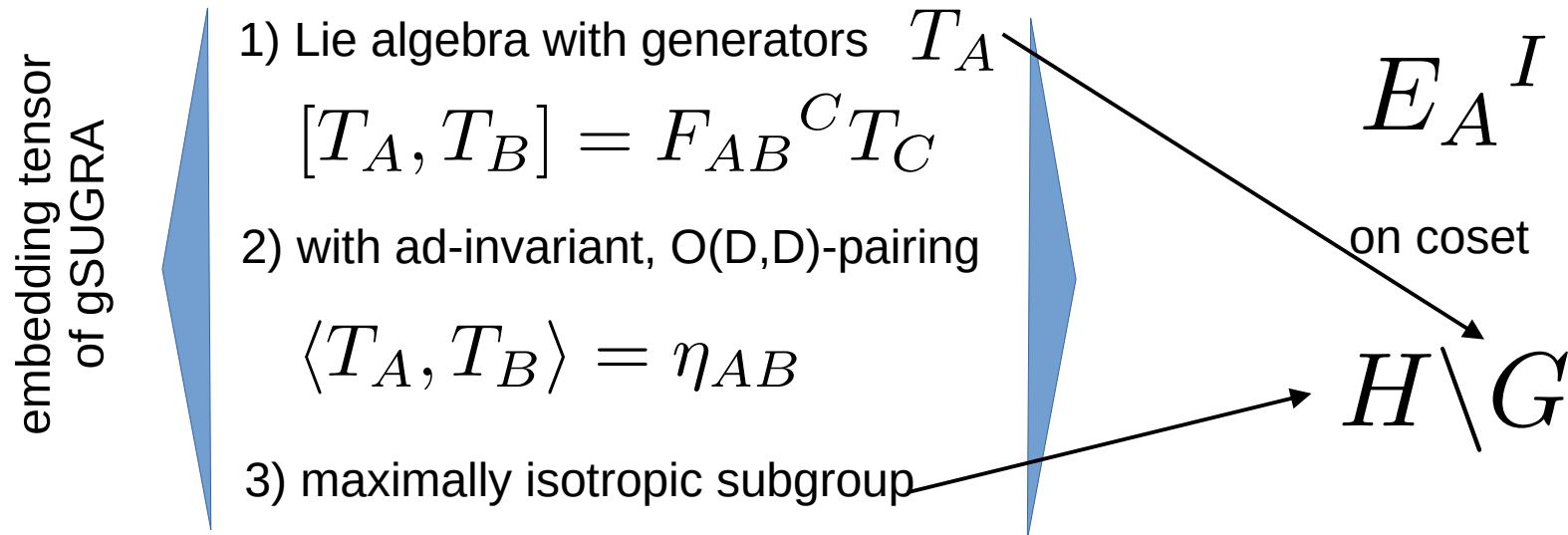
Generalized group manifold



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Homogeneous space

Theorem: Let (M,g) be a connected and simply-connected complete Riemannian manifold. Then, the following statements are equivalent:
[Ambrose, Singer 1958]

1) The manifold M is Riemannian homogenous

2) M admits a linear connection ∇ satisfying

$$\nabla R = 0, \quad \nabla S = 0, \quad \nabla g = 0$$

Riemann tensor \nearrow $S = \nabla^{\text{LC}} - \nabla$ \nwarrow metric

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 frame and connection required

$$\nabla_i e_a^j = \partial_i e_a^j - \omega_{ia}^b e_b^j + \Gamma_{ik}^j e_a^k = 0$$

Generalized coset

$$\nabla_I E_A^J = \partial_I E_A^J - \Omega_{IA}^B E_B^J + \Gamma_{IK}^J E_A^K = 0$$

O(D,D) recipe to construct gen. frame and spin connection:

- 1) gen. frame on $H \backslash G$ = mega-space
- 2) another isotropic subgroup F

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consistent
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E_A^I and Ω_{IA}^B

on double coset

$H \backslash G / F$

gen. structure group

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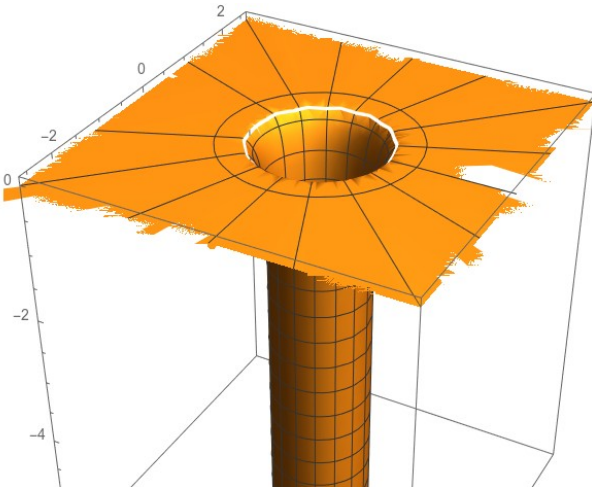
on double coset

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New

- *higher derivative* connections from *tensor hierarchy*
- *singularities @* fixed points of F action



Applications

worldsheet

current algebra

$$\{J_A(\sigma), J_B(\sigma')\} = F_{AB}{}^C J_C(\sigma)\delta(\sigma - \sigma') + \eta_{AB}\delta'(\sigma - \sigma')$$



generalized Lie derivative

All integrable string theories have gen.
coset/group target space *)

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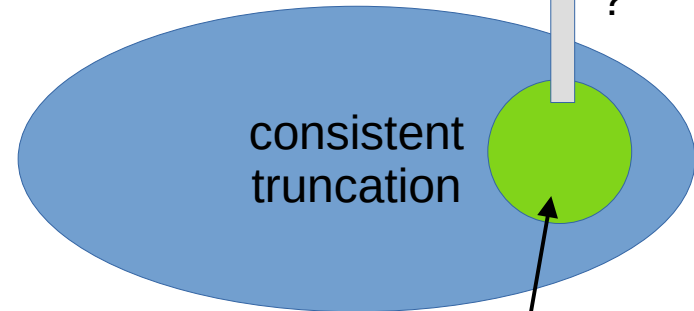


generalized Lie derivative

All integrable string theories have gen. coset/group target space *)

target space

gen. coset/group




> 2 derivatives



*) No exceptions known.

(Super)generalized geometries

$O(D,D)$:	$O(D, D)$	$E_{n(n)}, n \leq 8$	$OSp(D, D 2s)$
theory	bosonic	M & type IIB	type II $D=10,$ $s=32$
gen. group	✓	✓	✓
gen. coset	✓		✓
full spacetime	✓	✗	✓
fermions	✗	✗	✓
gen. duality	bosonic-T	U	super-T

Supergeneralized Lie derivative

$$\mathbb{L}_\xi V^{\mathcal{M}} = \xi^{\mathcal{N}} \partial_{\mathcal{N}} V^{\mathcal{M}} - V^{\mathcal{N}} \left(\partial_{\mathcal{N}} \xi^{\mathcal{M}} - \partial^{\mathcal{M}} \xi_{\mathcal{N}} (-)^{mn} \right)$$

leaves the $\text{OSp}(D,D|2s)$ metric

$$\eta_{\mathcal{M}\mathcal{N}} = \begin{pmatrix} 0 & \delta_{\mathcal{M}}^{\mathcal{N}} \\ \delta^{\mathcal{M}}_{\mathcal{N}} (-)^{mn} & 0 \end{pmatrix}$$

invariant

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closes under the section condition $\eta^{\mathcal{M}\mathcal{N}} \partial_{\mathcal{N}} \otimes \partial_{\mathcal{M}} = 0$ solved by

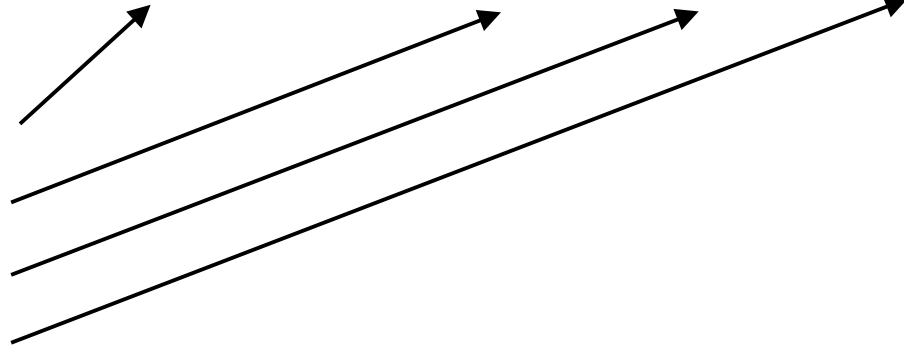
$$\partial_{\mathcal{M}} = \begin{pmatrix} \partial_M & 0 \end{pmatrix} \text{ with } \partial_M = \frac{\partial}{\partial z^M} \text{ and } z^{\mathcal{M}} = \begin{pmatrix} x^m & \theta^\mu \end{pmatrix}$$

Supergeneralized frame

$$\mathrm{OSp}(D, D|2s) \ni \mathcal{E}_{\mathcal{M}}^{\mathcal{A}} = (\mathcal{E}_B)_{\mathcal{M}}^{\mathcal{N}} \times (\mathcal{E}_E)_{\mathcal{N}} \times (\mathcal{E}_S)^{\mathcal{B}} \Lambda_{\mathcal{B}}^{\mathcal{A}}$$

contributions:

- 1) super B-field
- 2) super frame
- 3) R/R field strengths & dilatini
- 4) super double Lorentz $\mathbb{H}_L \times \mathbb{H}_R$



Supergeneralized frame

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New

$$V_{\mathcal{A}} = \left(\begin{array}{ccc|ccc} V_a & V_{\alpha} & V^{\alpha} & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{array} \right) \begin{array}{c} \text{left} \\ \text{right} \end{array}$$

generators of \mathbb{H}_L , $\lambda_{BA} = -\lambda_{BA}(-)^{ab}$ are further constraint by

$$\lambda_{\beta\alpha} = \lambda_{b\alpha} = 0, \quad \lambda_{\beta}^{\alpha} = \frac{1}{4} \lambda_{ba} (\gamma^{ba})_{\beta}^{\alpha}, \quad (\gamma^b)_{\beta\alpha} \lambda_b^{\alpha} = 0$$

Supergeneralized group manifold

on the super coset $H \backslash G \ni m$

1) one-forms $dm m^{-1} = V^A T_A + A_A T^A (-)^a$

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3) adjoint action $M_{\mathcal{A}}^{\mathcal{B}} = (\text{Adj } m)_{\mathcal{A}}^{\mathcal{B}} = \langle m T_{\mathcal{A}} m^{-1}, T^{\mathcal{B}} \rangle$

Supergeneralized coset

split G 's generators into $T_{\hat{A}} = \left(\begin{array}{ccc} \underline{T_A} & T_A & T^{\underline{A}} \end{array} \right)$ with
 subalgebra generators and their dual

$$\eta_{\hat{A}\hat{B}} = \begin{pmatrix} 0 & 0 & \delta_{\underline{A}\underline{B}} \\ 0 & \eta_{\mathcal{A}\mathcal{B}} & 0 \\ \delta^{\underline{A}\underline{B}}(-)^b & 0 & 0 \end{pmatrix} \quad \text{and double coset elements}$$

$n \in H \backslash G / F, \quad m = nf, \quad f \in F$

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→ frame is of the form:

spin connection

$$\hat{\mathcal{E}}_{\hat{A}}^{\hat{\mathcal{M}}} = (\text{Adj } f)_{\hat{A}}^{\hat{B}} \begin{pmatrix} \delta_{\underline{B}\underline{C}} & 0 & 0 \\ -\Omega_{\underline{B}\underline{C}} & \mathcal{E}_{\underline{B}}^{\mathcal{N}} & 0 \\ \rho^{\underline{BC}} - \frac{1}{2}\Omega^{\underline{B}\underline{C}} & \Omega^{\underline{B}\mathcal{N}} & \delta^{\underline{B}\underline{C}} \end{pmatrix} \begin{pmatrix} \tilde{v}_{\underline{C}}^{\underline{I}} & 0 & 0 \\ 0 & \delta_{\mathcal{N}}^{\mathcal{M}} & 0 \\ 0 & 0 & \tilde{v}_{\underline{I}}^{\underline{C}}(-)^i \end{pmatrix}$$

2-derivative connection $\tilde{v}^{\underline{A}}T_{\underline{A}} = df f^{-1}$

Torsion constraints

- $O(D,D)$:
- $F_{AB}{}^C$ only constraint by Bianchi identity
 - generalized metric fixes double Lorentz group

$OSp(D,D|2s)$:

- **no** generalized metric, $H_L \times H_R$?
- SUSY algebra only closes on-shell



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torsion constraints

κ -symmetry/
gen. SUGRA



$$F_{\alpha\beta\gamma} = F_{\alpha\beta\bar{\gamma}} = F_{\alpha\bar{\beta}\bar{\gamma}} = F_{\bar{\alpha}\bar{\beta}\bar{\gamma}} = 0$$

$$F_{\alpha\beta c} = -i\sqrt{2}(\gamma_c)_{\alpha\beta} , \quad F_{\bar{\alpha}\bar{\beta}c} = -i\sqrt{2}(\bar{\gamma}_c)_{\bar{\alpha}\bar{\beta}} ,$$

$$F_{\alpha\bar{\beta}c} = F_{\alpha\beta\bar{c}} = F_{\alpha\beta c} = F_{\bar{\alpha}\bar{\beta}c} = 0$$

η - and λ -deformation

$$\text{AdS}_5 \times \mathcal{S}^5$$



- worldsheet theory integrable
- target space max. SUSY



integrability
without
SUSY

?

$$\mathcal{G} = \text{PSU}(2, 2|4)$$

$$F = \text{SO}(4, 1) \times \text{SO}(5)$$

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integrability
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?

$$H \backslash \mathcal{G}^{\mathbb{C}} / F$$

$$H \backslash \mathcal{G} \times \mathcal{G} / F$$

$$\eta = \frac{1 - \lambda}{1 + \lambda}$$

η -deformation

PL T-duality

λ^* -deformation



analytic continuation

λ -deformation

only one parameter, η , allowed by torsion constraints

Summary and outlook

- unified all known T-dualities into one framework
- understand both,
 - worldsheet: Green-Schwarz string
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 - worldsheet: Green-Schwarz string
 - target space: generalized supergravity
- application:
 - integrable superstrings
 - new solutions from fermionic T-dualities
 - study fate for SUSY under generalized T-dualities
- new questions:
 - (How) do branes resolve singularities of generalized cosets?
 - Do higher derivative corrections result in equivalence

generalized dualities



consistent truncations