9.5. State operator correspondence

Remember: $\phi(z) = \sum_{n \in \mathbb{Z}} \phi_n z^{-n-h}$

$\langle \phi(z) \rangle$ = regular @ $\begin{cases} z = 0 & \hat{=} \quad \tau = -\infty \\ z = \infty & \hat{=} \quad \tau = \infty \end{cases}$

$\phi_n |0\rangle = 0$ for $n > -h$ annihilation op.

$\langle 0| \phi_n = 0$ for $n < h$ creation op.

$$|\phi_{in}\rangle = \lim_{z \to 0} \phi(z) |0\rangle = \phi(0)|0\rangle = \phi_{-h} |0\rangle$$

State operator correspondence

where $\phi_{-h} = \oint_{C_0} \frac{dz}{2\pi i} \frac{1}{z} \phi(z)$

Same applies for out-States:

$$\langle \phi_{out}| = \lim_{z \to \infty} \langle 0| \phi^\dagger(z) z^{2h} = \langle 0|(\phi^\dagger)_h$$

$\Rightarrow \langle \phi_i | \phi_j \rangle = \lim_{\substack{z \to 0 \\ w \to \infty}} w^{2h_j} \langle 0| \phi_j^\dagger(w) \phi_i(z) |0\rangle$

9.6. Vertex operators

Insert: How to compute OPE's effeciently?

$$:\phi_i(z) \phi_j(z): = \lim_{w \to z} \left(\phi_i(w) \phi_j(z) - \text{poles} \right) \checkmark$$

normal ordering OPE

$$= \frac{1}{2\pi i} \oint_{C_z} dw \frac{\phi_i(w) \phi_j(z)}{w - z}$$

Remember (again): $\phi_i(z) = \sum_{n \in \mathbb{Z}} \phi_{i,n} z^{-n-h_i}$

check $\underline{\underline{(:\phi_i \phi_j:)_m}} = \sum_{n \leq -h_i} \phi_{i,n} \phi_{j,m-n} + \sum_{n > -h_i} \phi_{j,m-n} \phi_{i,n}$

$\checkmark$ what we expect creation annihilation

Therefore: $\phi_i(z)\,\phi_j(w) = \underbrace{\phi_i(z)\,\phi_j(w)}_{} + :\phi_i(z)\,\phi_j(w):$

and $\langle \phi_i(z)\,\phi_j(w)\rangle = \underbrace{\phi_i(z)\,\phi_j(w)}_{}$

$\nearrow$ EX 8 $\quad \langle \partial X(z)\,\partial X(w)\rangle = -\frac{\alpha'}{2}\frac{1}{(z-w)^2}$

& remember

$V_k(z,\bar z) = :e^{ikX(z,\bar z)}:$ $\qquad T(z) = -\frac{1}{\alpha'} :\partial X(z)\,\partial X(z):$

Vertex op. $\qquad$ konstant

OPE: $\quad T(z)\,V_k(w,\bar w) = \left[\frac{\frac{\alpha'}{4}k^2}{(z-w)^2} + \frac{\partial_w}{(z-w)}\right] V_k(w,\bar w)$

similar for $\bar T(\bar z)$, check!

$\hookrightarrow$ $V_k(z,\bar z)$ is primary with $(h,\bar h) = \left(\frac{\alpha'}{4}k^2, \frac{\alpha'}{4}k^2\right)$

$$V_p(z,\bar z)\,V_q(w,\bar w) = |z-w|^{\alpha' p\cdot q}\, V_{p+q}(w,\bar w)$$

$$\partial X^\mu(z)\,V_k(w,\bar w) = -\frac{\alpha'}{2}\frac{ik^\mu}{z-w}V_k(w,\bar w) + \text{finite}$$

Why ??? State-operator correspondence !!!

$$|k\rangle = \lim_{z,\bar z\to 0} V_k(z,\bar z)\,|0\rangle$$

remember: $\quad p^\mu = \sqrt{\frac{2}{\alpha'}}\,\alpha_0^\mu = \frac{2}{\alpha'}\oint \frac{dz}{2\pi i}\, i\partial X^\mu$

$$p^\mu|k\rangle = \lim_{w,\bar w\to 0}\cancel{\frac{2}{\alpha'}}\oint\frac{dz}{2\pi i}\underbrace{i\,\partial X^\mu(z)\,V_k(w,\bar w)}_{\cancel{\frac{\alpha'}{2}}\frac{k^\mu}{z-w}V_k(w,\bar w)}|0\rangle$$

$$= k^\mu|k\rangle$$

Vacuum of string's Fock space

in general $\qquad$ primary

$$|\phi_i\rangle = \lim_{z,\bar z\to 0}\phi_i(z,\bar z)\,|0\rangle \quad \text{with}$$

$$L_0|\phi_i\rangle = h_i|\phi_i\rangle \quad\text{and}\quad \bar L_0|\phi_i\rangle = \bar h_i|\phi_i\rangle$$

physical state $(L_0-1)|\phi_i\rangle = (\bar{L}_0-1)|\phi_i\rangle = 0$

$\hookrightarrow (h,\bar{h}) = 1$ for $L_n|\phi\rangle = \bar{L}_n|\phi\rangle = 0$ for $n>0$

$$\boxed{\text{In CFT physical states are primary fields with conformal dimensions } (h,\bar{h})=(1,1).}$$

for $V_k(z,\bar{z}) \rightarrow m^2 = -k^2 = -\frac{4}{\alpha'}$ (tachyon)

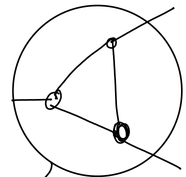
$$|\varepsilon,k\rangle = -\frac{2}{\alpha'}\varepsilon_{\mu\nu}\lim_{z,\bar{z}\to 0}\underbrace{:\partial X^\mu(z)\bar{\partial}X^\nu(\bar{z})\,V_k(z,\bar{z}):}_{\phi_{\varepsilon,k}(z,\bar{z})}|0\rangle$$

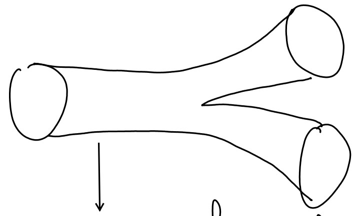
$$T(z)\,\phi_{\varepsilon,k}(z,\bar{z}) = -\frac{i\alpha'}{2}\frac{k^\mu\varepsilon_{\mu\nu}}{(z-w)^3}:\bar{\partial}X^\nu(\bar{w})\,V_k(w,\bar{w}): + \left[\frac{\frac{\alpha'}{4}k^2+1}{(z-w)^2}+\frac{\partial_w}{z-w}\right]\phi_{\varepsilon,k}(z,\bar{z})$$

$=0$ for primary, recovers polarisation condition

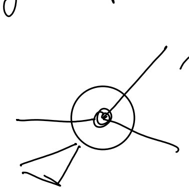
10. String perturbation theory

Motivation: compute probability of process



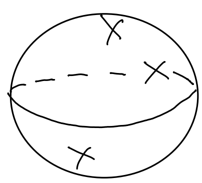


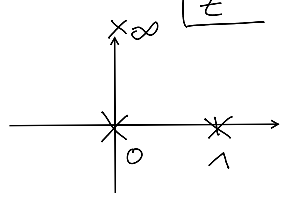
$\hat{=}$



Vertex in Feynman diagram

conformal transformation



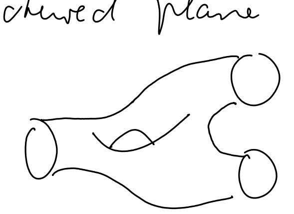


after Möbius transformation

three-punctured sphere

three-punctured plane

there are much more "diagrams" like



1) How do we enumerate them?

2) How they are computed?

10.1. String perturbation expansion

 Polyakov action $\hat{=}$ gravity with matter (X^μ's) in 2d

does not depend on world sheet metric, topological

↗ EX 4.2.

But we can compute <u>Euler number</u>,

$$\boxed{\chi = \frac{1}{4\pi}\int d^2\sigma \sqrt{h}\, R = 2 - 2g - b}$$

genus $\hat{=}$ number of holes

number of punctures on the worldsheet

← amplitude in closed string theory

$$A_n = \sum_{g=0}^{\infty} A_n^{(g)} = \sum_{g=0}^{\infty} C_{\Sigma_g} \int \mathcal{D}h\, \mathcal{D}X^\mu \int d^2 z_1 \dots d^2 z_n$$

"number of loops"

$$\underbrace{V_1(z_1, \bar{z}_1) \dots V_n(z_n, \bar{z}_n)}_{\text{punctures with Vertex operators inserted}} \cdot e^{-S[X,h]}$$

C_{Σ_g} = weight for each world sheet topology

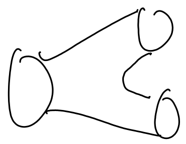
How to get C_Σ ? Add $-\lambda\chi$ to S_p !

Then $C_{\Sigma_g} \sim g_s^{-\chi} = g_s^{-(2-2g-b)}$ $\qquad g_s := e^{\lambda}$

not a new constant, but vev. of dynamical field $\lambda = \langle \phi \rangle$

<u>Remember</u> massless spectrum (sec. 5.7.)

$\boxed{\begin{matrix} 1 \\ \nearrow \\ \text{dilaton } \phi \end{matrix}}$ $\oplus$ sym. traceless $\hat{=} g_{\mu\nu}$ graviton $\oplus$ anti sym. $\hat{=} B_{\mu\nu}$ "B"- field

$g_s \hat{=}$ String coupling because  $\sim g_s$