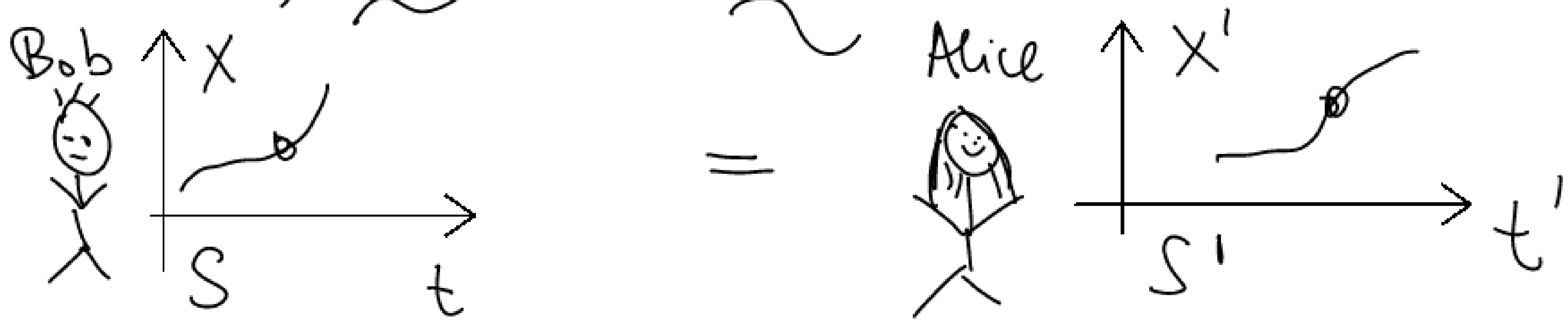
4.2. Physical interpretation

Experimental facts:

1. The laws of physics are the same in any inertial frame (principle of relativity)



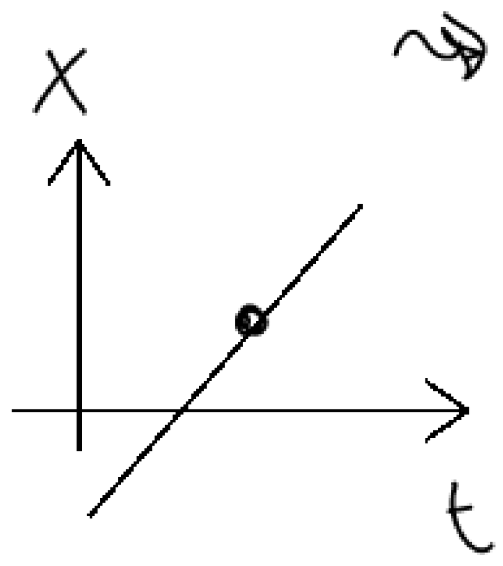
2. The speed of light is the same in all inertial frames

Question: Which transformation

$x' = f(x,t)$ & $t' = g(x,t)$

satisfies 1. and 2. ?

1. inertial frame, no external forces



$\rightsquigarrow$ particle will travel at const. velocity

$\rightsquigarrow$ lines are maped to lines

$\rightsquigarrow$ linear map $\quad x' = \alpha_1 x + \alpha_2 t$

$t' = \alpha_3 x + \alpha_4 t$

S' moves with velocity v relativ to S

$x = v \cdot t \quad \rightsquigarrow \quad x' = 0$ gives rise to

$$\boxed{x' = \gamma_v (x - vt)} \quad (1)$$

γ_v ... velocity dep. coeff.

rotational invariance $\quad |v| = \sqrt{\vec{v} \cdot \vec{v}}$

from the perspective of S'

$x' = -v \cdot t' \rightsquigarrow x = 0$

$$x = \gamma_v \left(x' + v \cdot t' \right) \qquad (2)$$

for absolute time $t = t'$ we would get

$\gamma_v = 1$ but 2. rather requires

$x = c \cdot t \rightsquigarrow x' = c \cdot t'$

$$\left.\begin{array}{ll} x' = c \cdot t' = \gamma_v \cdot (c - v) t & (1) \\ x = c \cdot t = \gamma_v (c + v) t' & (2) \end{array}\right\} \boxed{\gamma_v = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

Lorentz factor

and $\boxed{t' = \gamma \left(t - \frac{v}{c^2} x \right)}$ ~ Lorentz transformation.

check: for $v \ll c$ $\gamma_v \approx 1$ and

$t' = t$, $x' = x - v \cdot t$ (Galilean transformation)

In 4 dimensions:

Lorentz - boost

$$\boxed{\begin{array}{l} x' = \gamma_v \left(x - \frac{v}{c} c \cdot t \right) \\ y' = y, \quad z' = z, \quad \text{and} \\ ct' = \gamma_v \left(ct - \frac{v}{c} x \right) \end{array}}$$

Relation to last lecture? Compute:

$$c^2 t'^2 - x'^2 - y'^2 - z'^2 = x'^\mu x'^\nu \eta_{\mu\nu} =$$

$$c^2 t^2 - x^2 - y^2 - z^2 = x^\mu x^\nu \eta_{\mu\nu}$$

$$\rightsquigarrow \quad x'^\mu = L^\mu{}_\nu x^\nu$$

6 Lorentz transformations $\hat{=}$ 3 boost + 3 rotations

4.3. Phenomena

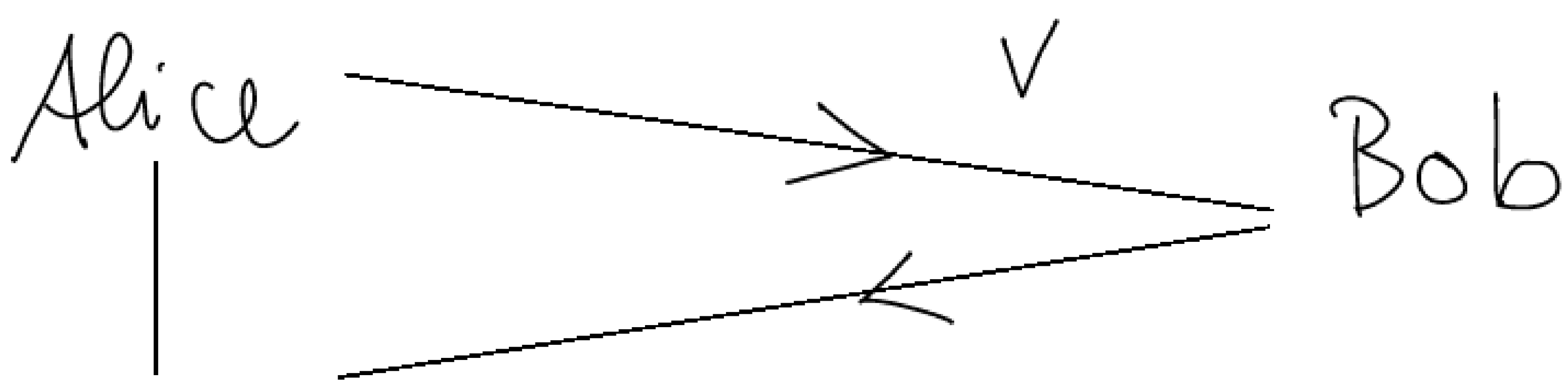
time dilation setup: clock in S'

with ticks @ $(c \cdot n \cdot T', 0)$ with inverse $ct = \gamma\left(ct' + \frac{v}{c}x'\right)$ we get

$$cnT = \gamma\left(cnT' + \frac{v}{c} \cdot 0\right)$$

$\leadsto$ $T = \gamma \cdot T'$ "moving clocks run slower"

"Twin paradox"



$$T_{Alice} = 2T > T_{Bob} = 2T/\gamma$$

But for Bob Alice moves. Why the asymmetry?

↗ EX 6

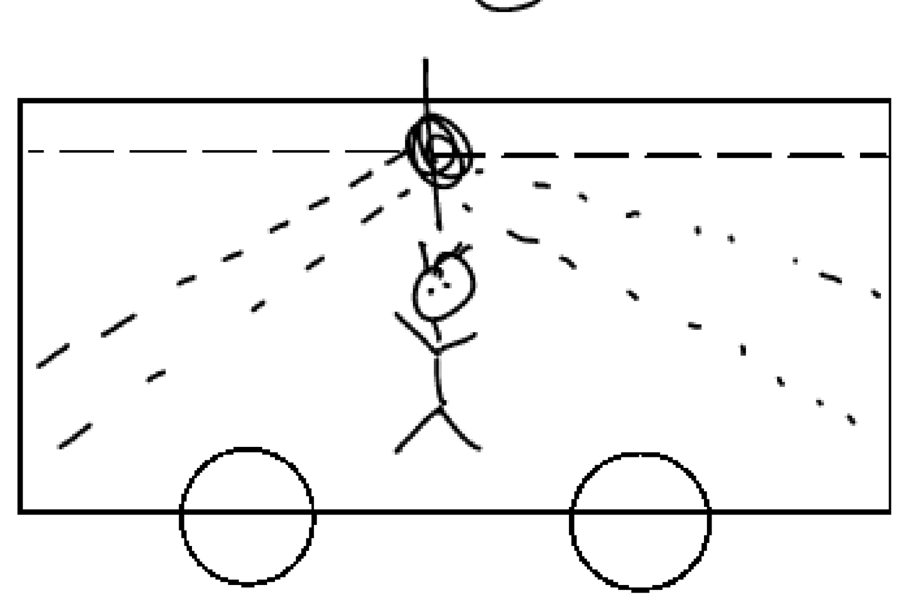
Simultaneity for absolute time $(t = t')$ easy

in SR only possible in each inertial frame

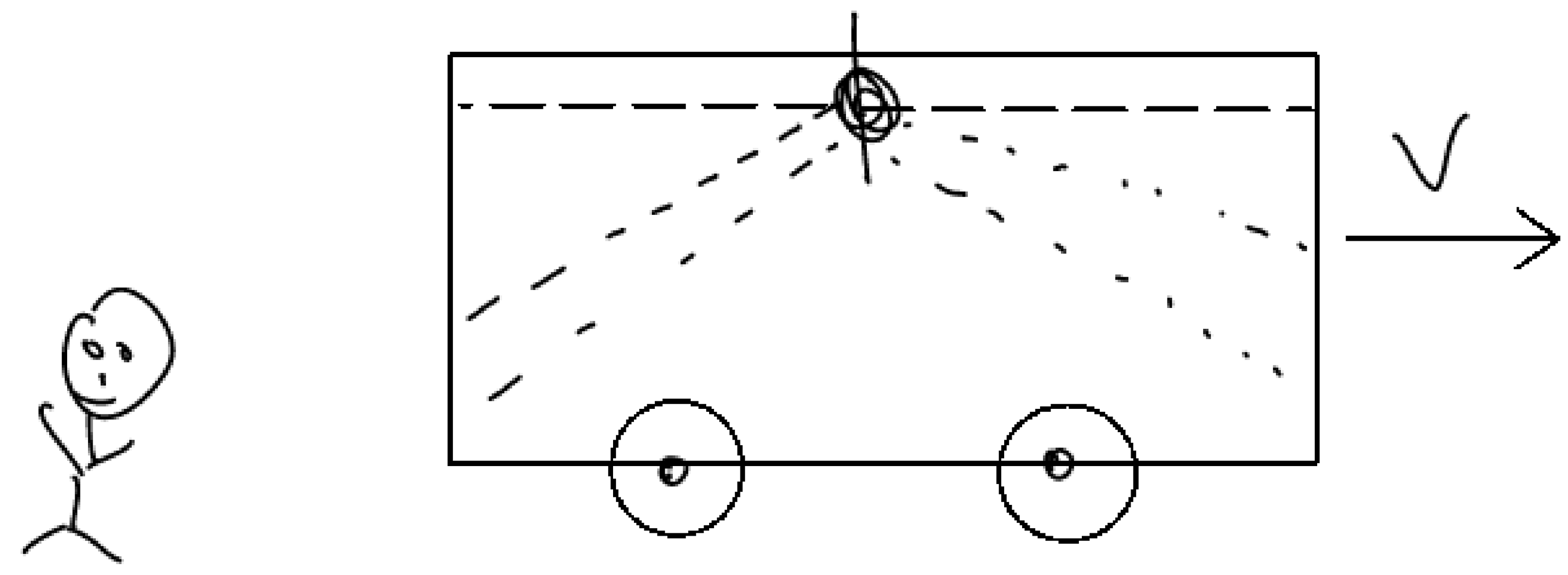
i.e. $t_1 = t_2 = t$

$$t'_{1/2} = \gamma\left(t - \frac{v}{c}x_{1/2}\right)$$

"Einstein train":

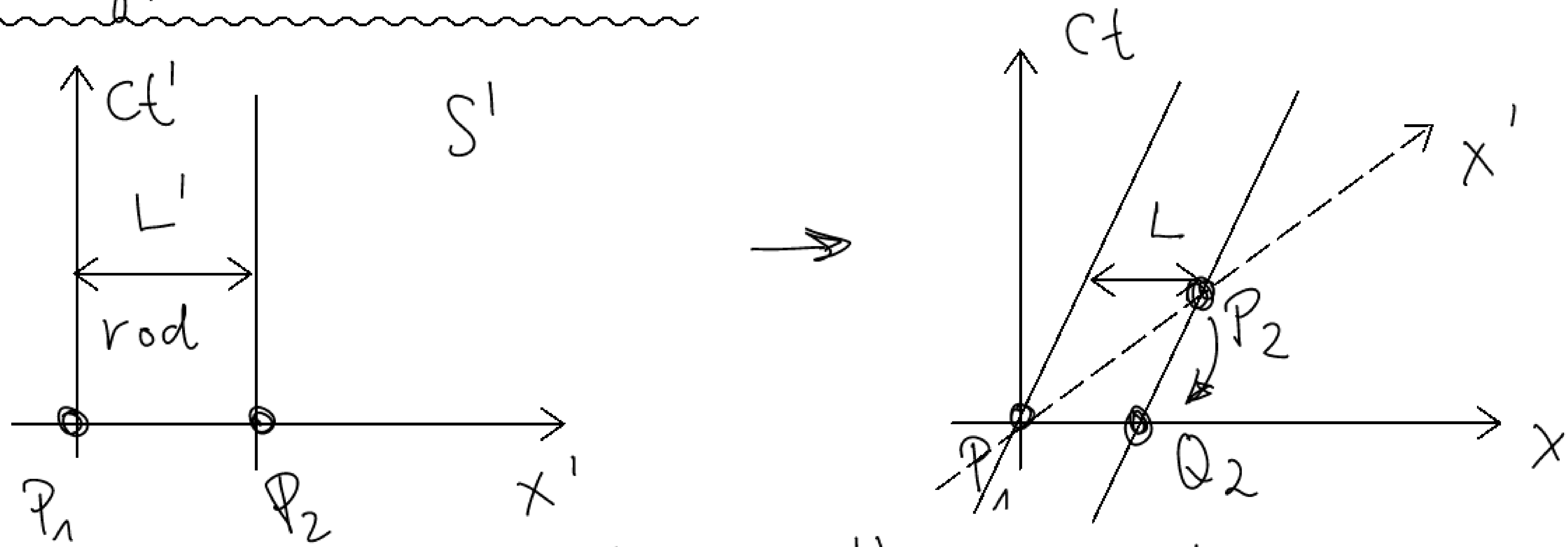


in car light hits front & back simultaneously



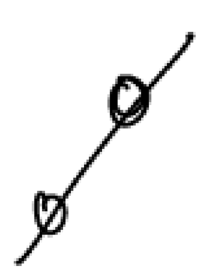
on platform, first back then front

length contraction



P_2: $\quad x = \gamma_v \left(x' + \frac{v}{c^2} \cdot t' \right) = \gamma_v L'$

$\qquad t = \gamma_v \left(t' + \frac{v}{c^2} x' \right) = \gamma_v \frac{v}{c^2} L'$



$\qquad x = \gamma L' - vt$

Q_2: $\qquad x = \gamma L' - \dfrac{\gamma v^2 L'}{c^2} = \dfrac{L'}{\gamma}$

$\rightsquigarrow L = \dfrac{L'}{\gamma}$ or "The length of the rod in the moving frame is shorter than in the rest frame."