☆ Classical Field Theory ☆

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lectures: Wed. 14:15 – 16:00 } 445
tutorials: Wed. 16:16 – 18:00 }

exercises, & handwritten notes at

https://www.fhassler.de/teaching/ws_25/clft

- online ~ 1 week before tutorial
- assigned at Fri. 21:00 –"–

Will be graded.

EXAM
- written exam @ end of semester
- at least 50% of the points of assigned problems to qualify

⇝ check your points at the website

Problems? contact me or Achilleas Gitsis at achilleas.gitsis@uwr.edu.pl

1. Introduction

classical / quantum > field theory

field : physical system with ∞-ly many degrees of freedom (dof)

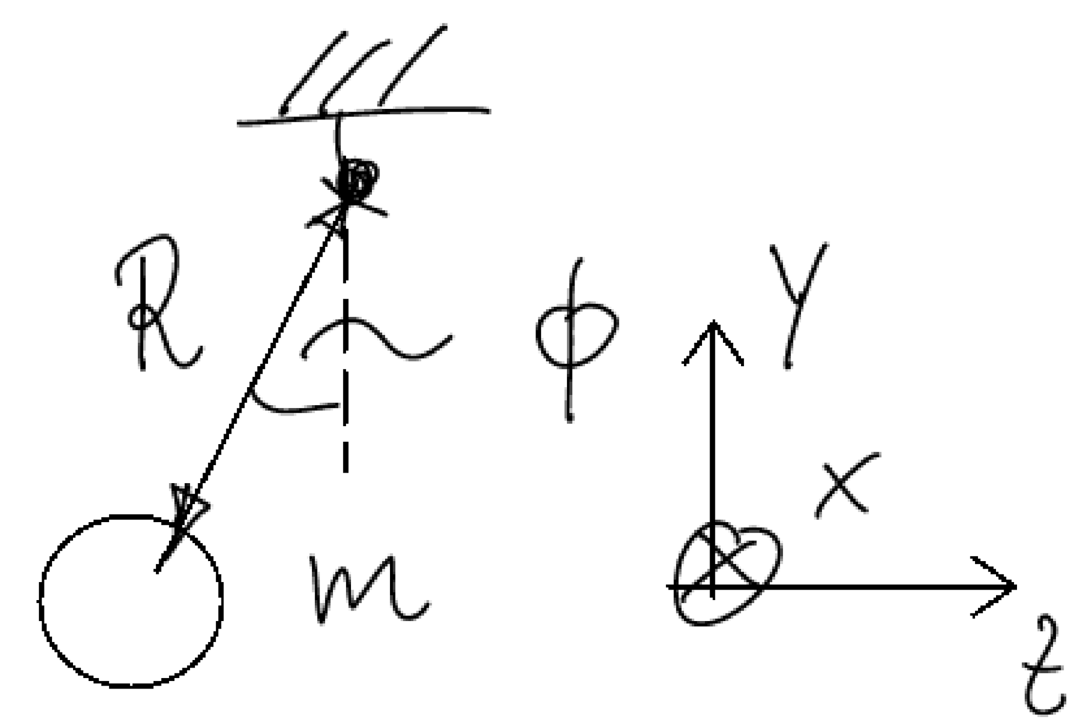
vs.

particle: — " — with finitely many dof ✓

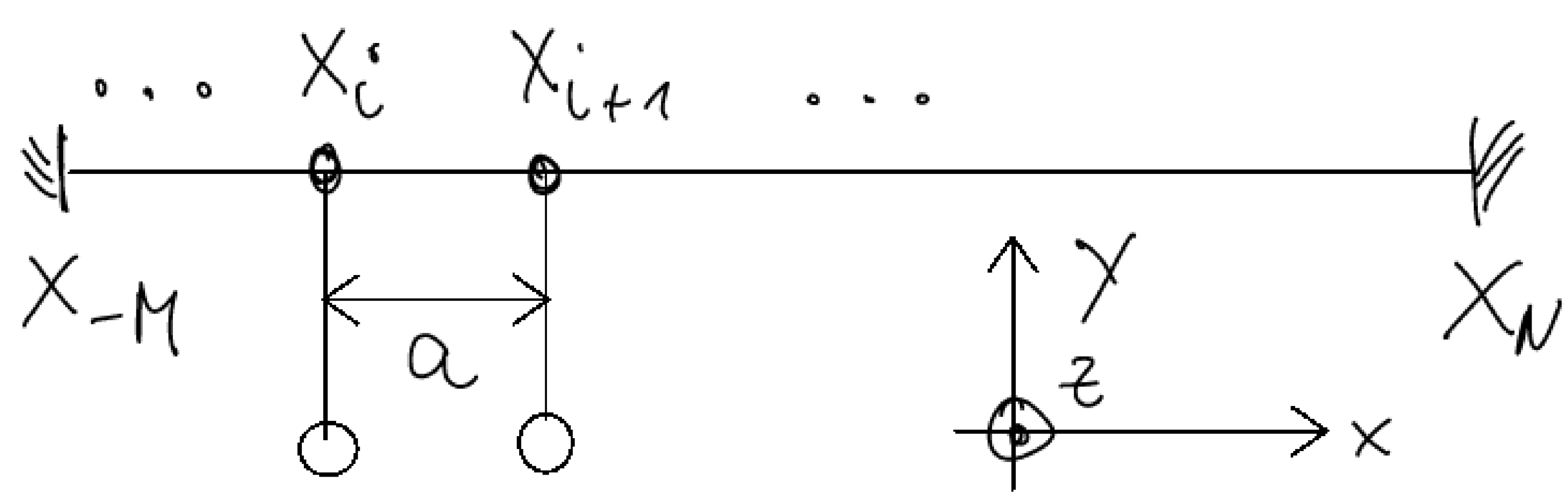
1.1. Sine-Gordon effective field

example 1 pendulum

or $M+N+1$ pendulums fixed on a wire at points



$x_i \quad i=-M,\dots,N$



equations of motion (eoms) $\quad \phi + O(\phi^3) \rightsquigarrow$ harmonic oscillator

$$\underbrace{m R^2 \frac{d^2\phi(x_i,t)}{dt^2}}_{\text{inertia}} = \underbrace{-mg R \sin \phi(x_i,t)}_{\text{gravitational force}} \tag{1}$$

$$+ \kappa \frac{\phi(x_i - a,t) - 2\phi(x_i,t) + \phi(x_i+a,t)}{a}$$

$$\left.\begin{aligned} \Delta\phi_L &= \phi(x_i - a,t) - \phi(x_i,t) \\ \Delta\phi_R &= \phi(x_i + a,t) - \phi(x_i,t) \end{aligned}\right\} \text{ at position } x_i$$

force from the wire $\left\{ = \kappa \dfrac{\Delta\phi_L + \Delta\phi_R}{a} \right.$

⚡ equations are missing $\Delta\phi_L$ at x_{-M} and $\Delta\phi_R$ at x_N

outer pendulums are fixed to $\phi(-x_M, t) = 0$ and $\phi(x_N, t) = 2\pi n$

Assuming initial conditions $\phi(x_i, t_0)$ and $\dot{\phi}(x_i, t_0)$ we can solve the eoms. $\to$ Very hard

Trick: continuum (fied theory) limit $\phi(x_i, t) \to \phi(x, t)$

$$\phi(x_i - a, t) - 2\phi(x_i, t) + \phi(x_i + a, t) = \int_0^a ds_1 \int_{-a}^0 ds_2 \left. \frac{\partial^2 \phi(s_1 + s_2 + x, t)}{\partial x^2} \right|_{x = x_i}$$

check

$$\int_0^a ds_1 \int_{-a}^0 ds_2 \frac{\partial^2 \phi(s_1 + s_2 + x, t)}{\partial s_1 \partial s_2} = \int_0^a ds_1 \left. \left(\frac{\partial \phi(s_1 + x, t)}{\partial s_1} - \frac{\partial \phi(s_1 - a + x, t)}{\partial s_1} \right) \right|_{x = x_i}$$

Assumption:

$$\boxed{\int_0^a ds_1 \int_{-a}^0 ds_2 \frac{\partial^2 \phi(s_1 + s_2 + x, t)}{\partial^2 x} \approx a^2 \left. \frac{\partial^2 \phi(x, t)}{\partial^2 x} \right|_{x = x_i}}$$

i.e. $\frac{\partial^2 \phi(x,t)}{\partial^2 x}$ nearly constant in $[x_i - a, x_i + a]$

With (1) we then get

$$m R^2 \frac{\partial^2 \phi(x,t)}{\partial^2 t} = - m g R \sin \phi(x,t) + \kappa a \frac{\partial \phi(x,t)}{\partial x^2}$$

with the boundary conditions $\phi(-Ma, t) = 0$

$\phi(Na, t) = 2\pi n$

Sine-Gordon equation

after substitutions $\tau = \sqrt{\frac{g}{R}}\, t$, $\xi = \sqrt{\frac{mgR}{\kappa a}}$, $\Phi(\xi, \tau) = \phi(x, t)$

$$\boxed{\frac{\partial^2 \Phi(\xi,\tau)}{\partial \tau^2} - \frac{\partial^2 \Phi(\xi,\tau)}{\partial \xi^2} + \sin \Phi(\xi,\tau) = 0} \quad (2)$$

check in EX 1.1 a)

with boundary conditions $\Phi(\xi_{-M},\tau) = 0$

$\Phi(\xi_N,\tau) = 2\pi n$

(2) is a partial differential equation (PDE)

finding solution $\Phi(\xi,\tau)$ requires initial conditions

$\Phi(\xi,\tau_0)$ and $\left.\frac{\partial \Phi(\xi,\tau)}{\partial \tau}\right|_{\tau=\tau_0}$ for fixed time τ_0

You will analyze solutions of (2) in EX 1.1.