14. Grant Unified Theories

remember, last time we packaged the first generation standard model particles into irreps of

$$\boxed{su(3) \times su(2) \times u(1)}$$

1) right-handed

$$(3,1)_{2/3} \oplus (3,1)_{-1/3} \oplus (1,1)_{-1} \oplus (\bar{3},2)_{-1/6} \oplus (1,2)_{1/2}$$

u^+ (up quark) d^+ (down quark) e^+ (electron) $\bar{\psi}^+ = \begin{pmatrix} \bar{d}^+ \\ \bar{u}^+ \end{pmatrix}$ } anti-quarks $\bar{\ell}^+ = \begin{pmatrix} \bar{e}^+ \\ \bar{\nu}^+ \end{pmatrix}$ } anti-leptons

2) left-handed

in the complex conjugate representation of 1):

$$\underbrace{(\bar{3},1)_{-2/3} \oplus (\bar{3},1)_{1/3} \oplus (1,1)_{1}}_{\text{anti-particles}} \oplus (3,2)_{1/6} \oplus (1,2)_{-1/2}$$

ψ^+ (quarks) ℓ^+ (leptons)

Question: can these particles originate from a bigger group $G \supset su(3) \times su(2) \times u(1)$ which is spontaneously broken?

14.1. su(5)

Constraints: $\text{rank } G \geq \text{rank}(su(3) \oplus su(2) \oplus u(1)) = 4$

i.e. $G = su(5)$

branching rule form section 12.5:

$$5 \to (3,1)_2 \oplus (1,2)_{-3} \to \text{rescale } u(1)\text{-generator}$$

$$5 \to (3,1)_{-1/3} \oplus (1,2)_{1/2} \qquad (\cdot - 1/6$$

We can now also compute the branching:

$$\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} = 10 \to (\bar{3},1)_{-2/3} \oplus (1,1)_1 \oplus (3,2)_{1/6}$$

$\rightsquigarrow$ 1) right-handed & 2) left-handed

$5 \oplus \overline{10}$ $\qquad$ $\bar{5} \oplus 10$

Breaking the $su(5)$ to $su(3) \times su(2) \times u(1)$ by the Higgs-mechanism requires:

1) Higgs field in the adjoint

$$24 \to (1,1)_0 \oplus (1,3)_0 \oplus (8,1)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{5/6}$$

U(1)-adj SU(2)-adj SU(3)-adj

give vacuum expectation value

2) Masses for fermions through Yukawa-couplings

coupling $\bar{\Psi}\Psi \phi$ ← Higgs field

particle anti-particle

positron $\bar{e}$ and d-quark d in the 5

electron e and anti-d-quark $\bar{d}$ in the $\overline{10}$

$$\hookrightarrow \quad 5 \otimes \overline{10} = \bar{5} \oplus \overline{45}$$

branchings:
$$\bar{5} \to (\bar{3},1)_{1/3} \oplus \boxed{(1,2)_{-1/2}}$$
$$45 \to \boxed{(1,2)_{-1/2}} \oplus \dots$$

← correct irreps for standard-model Higgs-field

right-handed u- and anti-u-quark are both in the $\overline{10}$, therefore:
$$\overline{10} \otimes \overline{10} = 5 + 45 + 50$$

no $(1,2)$ contribution for Higgs

Problem: $\overline{10}$ contains:
- electron $(1,1)_{-1}$
- anti-quarks $(\bar{3},2)_{-1/6}$
- quarks $(3,1)_{2/3}$

⇝ some SU(5) interactions do not conserve baryon number

$$B = \frac{1}{3}(n_q - n_{\bar{q}})$$

n_q number of quarks
$n_{\bar{q}}$ number of anti-quarks

⟶ proton decay! Yet, not observed in nature.

14.2. SO(10)

So far only consider particles we know from the SM. But, there could be more, very massive ones.

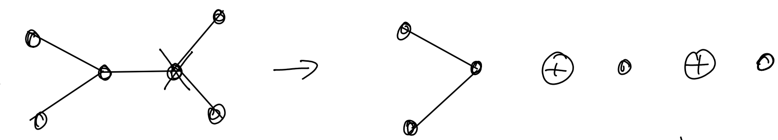
i.e. take the spinor irrep, 16, of SO(10) and branch to SU(5)

$16 \to 5 + \overline{10} + 1$ and additional particle, should get large mass

$\overline{16} \to \overline{5} + 10 + 1$

Interesting: all right-/left-handed particles of one gen. in one irrep.

↯ SU(5) is not a maximal regular subgroup

$\hat{D}_5$: 

$= su(4) \oplus su(2) \oplus su(2)$

contains color su(3)

mirror image of weak interaction

with branchings:

$16 \to (4,1,2) \oplus (\overline{4},2,1)$

$\overline{16} \to (\overline{4},1,2) \oplus (4,2,1)$

this mirror image has to be broken to restore CP-violation of the weak interaction observed in nature.

breaking the SO(10) to SU(5)

Yukawa couplings: $\overset{\Psi}{16} \otimes \overset{\Psi}{16} = 10 \oplus 120 \oplus 126$

only the branching of

$126 \to 1 \oplus 5 \oplus \overline{10} \oplus 15 \oplus \overline{45} \oplus 50$

contains singlet

Choose a Higgs field that couples to this singlet does not generate mass terms for the 5 or $\overline{10}$, just for the 1. ≙ right-handed neutrino

adjoint representation:

$45 \to 24 \oplus \overline{10} \oplus 10 \oplus 1$

only VEV for 45 gives rise to an unwanted

$SU(3) \times SU(2) \times U(1)^{2}$

broken by additional VEV of 16 or 126.

breaking SO(10) to $SU(4) \times SU(2) \times SU(2)$

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done by VEV of 54 and VEV of 16 or 126

break further to $SU(3) \times SU(2) \times U(1)$