13. Application: Particle Physics

remember original motivation: describe symmetries

more precise: action $S = \int d^4x \, \mathcal{L}(\phi_i)$

Lagrangian ($\mathcal{L}$)

degrees of freedom, i.e. fields (ϕ_i)

symmetry: transformation $\phi_i \to \Lambda_i{}^j(x)\,\phi_j$
that does not change the action

check: 1) infinitesimal transformation
$$\delta_\lambda \phi_i = \lambda_i{}^j(x)\,\phi_j$$
very small ($\lambda_i{}^j$)

2) What happens for two such transformations?
$$\delta_{\lambda_2}\delta_{\lambda_1}\phi - \delta_{\lambda_1}\delta_{\lambda_2}\phi = \delta_{\lambda_3}\phi \quad \text{(closure)}$$

$$\lambda_3 := [\lambda_1, \lambda_2] \longleftarrow \text{Lie bracket}$$

symmetries $\longleftrightarrow$ Lie algebras

global symmetry	local (gauge) symmetry
• $\lambda_i{}^j$ does not depend on the coordinates x^μ	• $\lambda_i{}^j(x)$ depends on the coordinates • introduces new fields, gauge potential, whose excitations are force carrier

Examples:

• Lorentz symmetry	• standard model
$so(3,1;\mathbb{R}) \hookrightarrow so(3,1;\mathbb{C})$ $\hat{=}\, su(2) \oplus su(2)$	$\underbrace{SU(3) \times SU(2) \times U(1)}$, organises the particle content
$(1,1)$: scalar $(2,1)$: left-handed Weyl-spinor	i.e. u: up quark d: down – " –

(1,2): right-handed -"-

(2,2): 4-vector

...

e: electron

ν: neutrino

13.1. Glashow-Salam-Weinberg theory

$su(2) \times u(1)$ Lie algebra describes the electroweak

gen. by R_a and S ≙ electromagnetic force

$a = 1, 2, 3$ ≙ weak nuclear force

take now the creation operators:

$$\underbrace{u^+, d^+, e^+}_{\text{right-handed}}, \underbrace{\bar{u}^+, \bar{d}^+, \bar{e}^+, \bar{\nu}^+}_{\text{left-handed}}$$

$$l_r^+ = \begin{pmatrix} l_1^+ \\ l_2^+ \end{pmatrix} = \begin{pmatrix} \bar{e}^+ \\ \bar{\nu}^+ \end{pmatrix} \qquad \Psi_r^+ = \begin{pmatrix} \Psi_1^+ \\ \Psi_2^+ \end{pmatrix} = \begin{pmatrix} \bar{d}^+ \\ \bar{u}^+ \end{pmatrix}$$

split into two $su(2)$ doublets

symmetry acts as:

1) on right-handed particles:

$$\left.\begin{aligned} [R_a, u^+] &= 0 \\ [R_a, d^+] &= 0 \\ [R_a, e^+] &= 0 \end{aligned}\right\} \text{trivial under } su(2) \qquad \begin{aligned} [S, u^+] &= \tfrac{2}{3} u^+ \\ [S, d^+] &= -\tfrac{1}{3} d^+ \\ [S, e^+] &= -e^+ \end{aligned}$$

2) on left-handed particles:

$$\left.\begin{aligned} [R_a, \Psi_r^+] &= \tfrac{1}{2} \Psi_s^+ \sigma^a_{sr} \\ [R_a, l_r^+] &= \tfrac{1}{2} l_s^+ \sigma^a_{sr} \end{aligned}\right\} \text{both } su(2) \text{ doublets} \qquad \begin{aligned} [S, \Psi_r^+] &= -\tfrac{1}{6} \Psi_r^+ \\ [S, l_r^+] &= \tfrac{1}{2} l_r^+ \end{aligned}$$

Pauli-matrices, generate the 2 of su(2)

electrical charge: $Q = R_3 + S$ with

$$[Q, u^+] = \tfrac{2}{3} u^+ \qquad [Q, e^+] = -e^+$$

$$[Q, d^+] = -\tfrac{1}{3} d^+ \qquad [Q, \bar{\nu}] = 0.$$

- we found the charges of the first gen. fermions ❗
- R_a and S represent gauge bosons that mediate the interaction

$$W^{\pm} \hat{=} R_1 \pm i R_2 \quad \Big\} \text{ weak-interaction}$$
$$Z = R_3 - S$$

W & Z bosones

$$P = R_3 + S \quad \Big\} \text{ electro-magnetic-interaction}$$

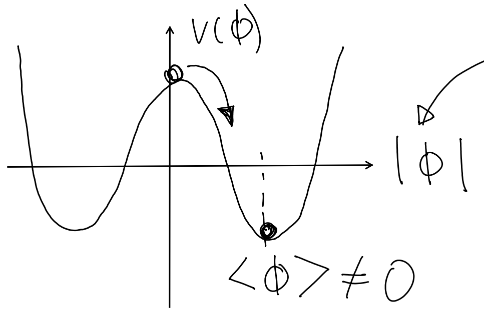
Photon

⚡ weak-interaction is much different from em.

i.e. short-ranged vs. long ranged

→ This symmetry is spontaneously broken in nature by the Higgs mechanism.

Idea: Vacuum of theory $|0\rangle$ is just invariant under $R_3 + S$ but not the other 3 generators.



Higgs field (su(2) doublet)

Instable configuration $\langle \phi \rangle = 0$ with unbroken symmetry decays to vacuum with $\langle \phi \rangle \neq 0$ and broken symmetry.

13.2. Adding color

full gauge group $SU(3) \times SU(2) \times U(1)$ with creation op. $a^{+}_{y,r}$ (new color index: y) in

$$\boxed{\begin{aligned} &\text{irrep } (D, d)_s \\ &[T_i, a^{+}_{x,r}] = a^{+}_{y,r} (T_i^D)_{yx} \\ &[R_a, a^{+}_{x,r}] = a^{+}_{x,s} (R_a^d)_{sr} \\ &[S, a^{+}_{x,r}] = s\, a^{+}_{x,r} \end{aligned}}$$

and we find:

right handed particle

$u^+ : (3,1)_{2/3}$, $d^+ : (3,1)_{-1/3}$, $e^+ : (1,1)_{-1}$

left handed particle

$\psi^+ : (\bar{3},2)_{-1/6}$, $\ell^+ : (1,2)_{1/2}$

re cap: