con't Higg's mechanism

last time SED with $V(\Phi) = -\frac{\mu^2}{2}\Phi\bar{\Phi} + \frac{\lambda}{4}(\Phi\bar{\Phi})^2$

and $\Phi(x) = \rho e^{i\theta}$

expand Lagrangian around minimum $\rho_0 = \frac{\mu}{\sqrt{\lambda}}$ of V

$$A_\mu = 0 + \delta A_\mu \,, \quad \rho = \frac{\mu}{\sqrt{\lambda}} + \delta\rho \,, \quad \theta = \underbrace{\theta_0}_{\text{constant!}} + \delta\theta$$

with $\boxed{B_\mu = \delta A_\mu + \frac{1}{e}\partial_\mu \delta\theta}$ gauge transformation with param. $\delta\theta$

upto quadratic order

$$\mathcal{L} = -\frac{1}{4}(2\partial_{[\mu}B_{\nu]})(2\partial^{[\mu}B^{\nu]}) \boxed{-\frac{1}{2}\frac{(\mu e)^2}{\lambda}B_\mu B^\mu}$$

$$-\frac{1}{2}\partial_\mu\delta\rho\,\partial^\mu\delta\rho - \frac{1}{2}\lambda^2\delta\rho^2$$

We find: 1. Massive scalar spin 0 field $\delta\rho$, like for global symmetries

2. Massive vector spin 1 B_μ, governed by

Proca Lagrangian:

$$\boxed{\mathcal{L}_{Proca} = -\frac{1}{4}(2\partial_{[\mu}B_{\nu]})(2\partial^{[\mu}B^{\nu]}) - \frac{1}{2}\varkappa^2 B_\mu B^\mu}$$

e.o.m: $= -\frac{1}{2}(\partial_\mu B_\nu \partial^\mu B^\nu - \partial_\mu B_\nu \partial^\nu B^\mu + \varkappa^2 B_\mu B^\mu)$

$$(\partial_\mu\partial^\mu - \varkappa^2)B_\nu = 0 \qquad \partial^\mu B_\mu = 0 \text{ constraint}$$

Klein-Gordon equation for each component of B_μ

$\underbrace{4}_{\text{components}} - \underbrace{1}_{\text{constraint}} = 3$ independet, massive scalar fields $\Rightarrow$ furnish representation of <u>Poincaré group's</u> little group $SO(3)$

↗ sec. 4.4.

the massless photon has one more constraint from gauge fixing $\qquad 4 - 2 = \underbrace{2}_{\text{rep. of massless little group } SO(2)}$ independent fields

$\delta\theta$ ("Goldstone boson") is replacing it and introduces the longitudinal mode required for massive spin 1

Excursion: Yukawa potential

remember $A^0 = \phi$ $\quad$ same for Proca field

for a <u>static potential</u> $\phi(\vec{x}) = B^0(\vec{x})$ we get

e.o.m $\quad (\nabla^2 - \kappa^2)\,\phi(r) = \boxed{-\,\rho(r)}$ source term from j^0

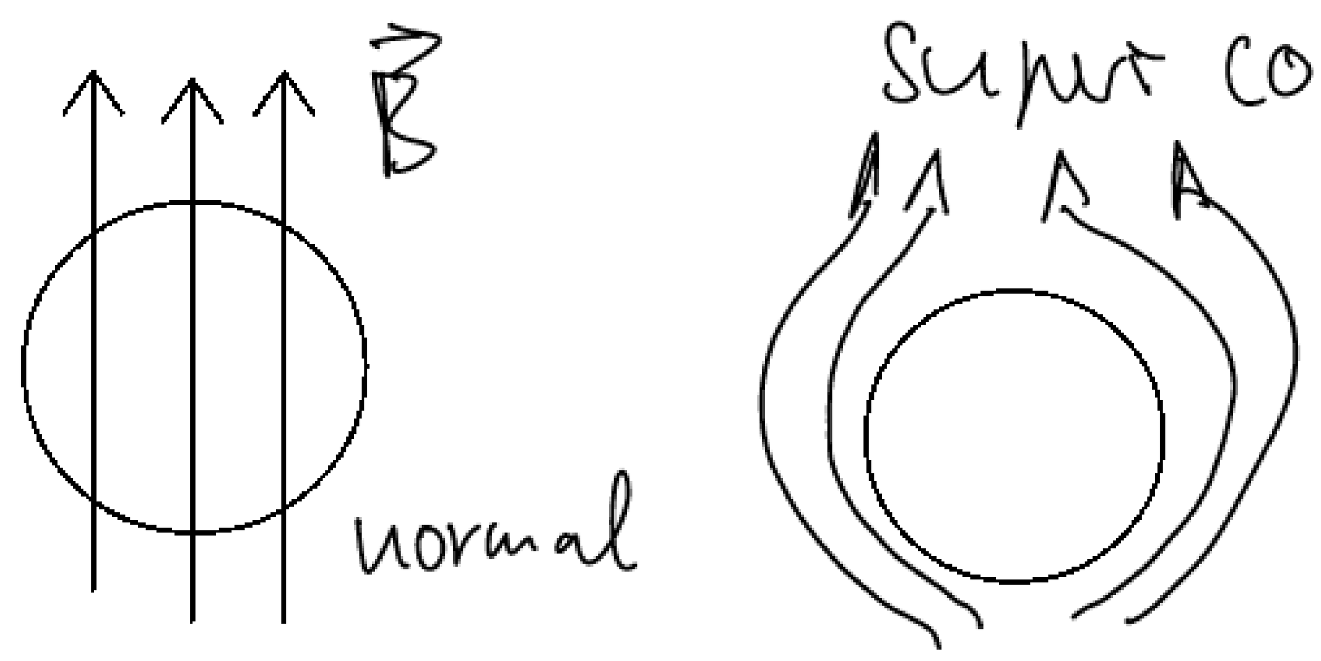
↗ sec. 5.2.

for a point source $\rho(r) = q\,\delta(r)$ the solution is

$$\boxed{\phi(r) = \frac{q}{4\pi r}\, e^{-\kappa r}}$$

Yukawa-potential with interaction length $\frac{1}{\kappa}$

one application: London penetration depth in super conductor



$$\nabla^2 B = \frac{1}{\lambda^2} B$$

↗ Landau-Ginzburg-theory

10. Dirac-field

We already know: representations of Poincaré group $\sim$

Spin	massless	massive
0		Klein-Gordon
1/2		Dirac
1	Maxwell	Proca

10.1. Spinors and γ-matrices

$$\Psi = \begin{pmatrix} \Psi_1 \\ \vdots \\ \Psi_4 \end{pmatrix} \in \mathbb{C}^4$$ is a spinor. How does Lorentz act?

Answer: Through γ-matrices! $\gamma_\mu : \mathbb{C}^4 \to \mathbb{C}^4$, $\mu = 0, 1, 2, 3$

with $$\boxed{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu = 2 \eta_{\mu\nu} \mathbb{1}_{4\times 4}}$$ Clifford algebra

Which matrices "solve" the Clifford algebra?

$$\gamma_0 = \begin{pmatrix} i\mathbb{1}_{2\times2} & 0 \\ 0 & -i\mathbb{1}_{2\times2} \end{pmatrix}, \quad \gamma_j = \begin{pmatrix} 0 & -i\sigma_j \\ i\sigma_j & 0 \end{pmatrix} \quad j = 1, 2, 3$$

with Pauli matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

They additionally satisfy ($\gamma \rightsquigarrow$ Dirac-matrices)

$$\gamma_0^\dagger = -\gamma_0 = \gamma_0^{-1}, \quad \text{and} \quad \gamma_i^\dagger = \gamma_i = \gamma_i^{-1}$$

(unique up to $\gamma'_\mu = U^{-1} \gamma_\mu U$ with $U^\dagger = U^{-1}$)

10.2. Dirac equation

$$\boxed{\gamma^\mu \partial_\mu \Psi + m \Psi = 0}$$

first order, linear PDE (4, coupled equations) with $\gamma^\mu = \eta^{\mu\nu}\gamma_\nu$, and $\partial_\mu \Psi = \begin{pmatrix} \partial_\mu \Psi_1 \\ \vdots \\ \partial_\mu \Psi_4 \end{pmatrix}$

one interpretation: Dirac $= \sqrt{\text{Klein-Gordon}}$ or

Dirac^2 = Klein-Gordon

$$\gamma^\mu \partial_\mu \Psi + m\Psi = 0 = \left(\gamma^\mu \partial_\mu + m I\right)\Psi$$

$$\left(\gamma^\mu \partial_\mu + m\Psi\right)^2 \Psi = \Big(\underbrace{\gamma^\mu \gamma^\nu}_{\mathbb{1}\cdot\eta^{\mu\nu}} \partial_\mu \partial_\nu + 2m\gamma^\mu \partial_\mu + m^2\Big)\Psi$$

(for any solution $\gamma^\mu \partial_\mu \Psi = -m\Psi$ we find

$$= \left(\partial_\mu \partial^\mu - m^2\right)\Psi = 0$$ Klein-Gordon :-)