9. Spontaneous symmetry breaking

remember:

$\boxed{\text{symmetries in QFTs}}$

global symmetries	local (gauge) symmetries

- continuous: Lie group → Abelian $U(1), \ldots$; non-Abelian $SU(2), \ldots$
- discrete

symmetries can be hidden, i.e.

$$\boxed{\begin{array}{l}\text{standard model}\\ \hat{=} \text{ gauge theory}\end{array}} \xrightarrow{\text{described}} \boxed{\begin{array}{l}\text{class}\\ \text{room}\end{array}}$$

Poincaré invariant law ~ theory $\xrightarrow[]{\text{symmetry breaking}}$ not Poincaré invariant state ~ solution (vaccum)

9.1. Global symmetries & Goldstone's Theorem

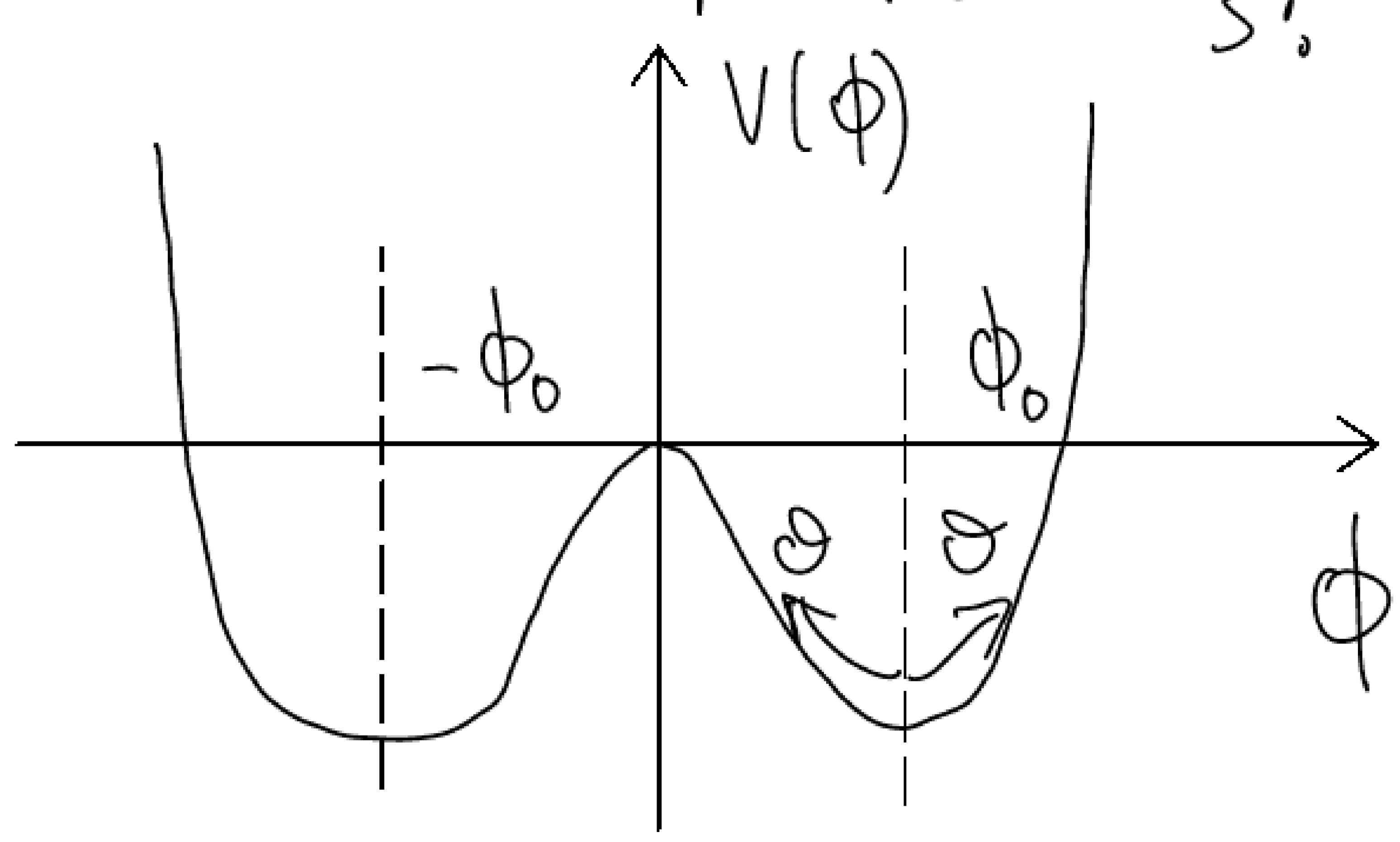
Example:
$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu \phi \partial^\mu \phi}_{= \;\; T} + \underbrace{\frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4!}\phi^4}_{-V}$$

we introduced "imaginary" mass $m = i\mu$ in ϕ^4 theory

e.o.m.
$$\partial_\mu \partial^\mu \phi - \mu^2 \phi + \frac{\lambda}{3!}\phi^3 = 0$$

for $\phi = \phi_0 = \text{const}$ $\quad -\mu^2 \phi_0 + \frac{\lambda}{3!}\phi_0^3 = 0$ or

$$\frac{dV}{d\phi} = 0$$



$$\phi_0 = \sqrt{\frac{6}{\lambda}}\,\mu$$

minimum of potential

Question: Why are there two solutions (ϕ_0 & $-\phi_0$)?

$\mathbb{Z}_2$ Symmetry of theory $\mathcal{L}(\phi) = \mathcal{L}(-\phi)$

To find more solutions, we expand around one solution, i.e. $\phi_0 \rightsquigarrow \boxed{\phi(x) = \phi_0 + \vartheta(x)}$

$$\mathcal{L}(\phi) = \tfrac{1}{2}\partial_\mu\vartheta\,\partial^\mu\vartheta \underbrace{- \tfrac{1}{2}(2\mu^2)\vartheta^2}_{\text{correct mass term with } m=\sqrt{2}\mu} \underbrace{- \sqrt{\tfrac{\lambda}{6}}\,\mu\,\vartheta^3}_{\text{no } \mathbb{Z}_2\text{-symmetry}} - \frac{\lambda}{4!}\vartheta^4$$

$\vartheta \leftrightarrow -\vartheta$ is not a symmetry anymore

Continuous symmetries: $\rightsquigarrow$ add fields to example

$$\mathcal{L} = \sum_{i=1}^{N}\left[\tfrac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i + \tfrac{1}{2}\mu^2(\phi^i)^2\right] - \frac{\lambda}{4}\left[\sum_{i=1}^{N}(\phi^i)^2\right]^2$$

with global $O(N)$-symmetry

$$\phi^{i\prime} = \sum_{j=1}^{N} R^i{}_j\,\phi^j \quad \text{or} \quad \vec{\phi}' = R\cdot\vec{\phi}$$

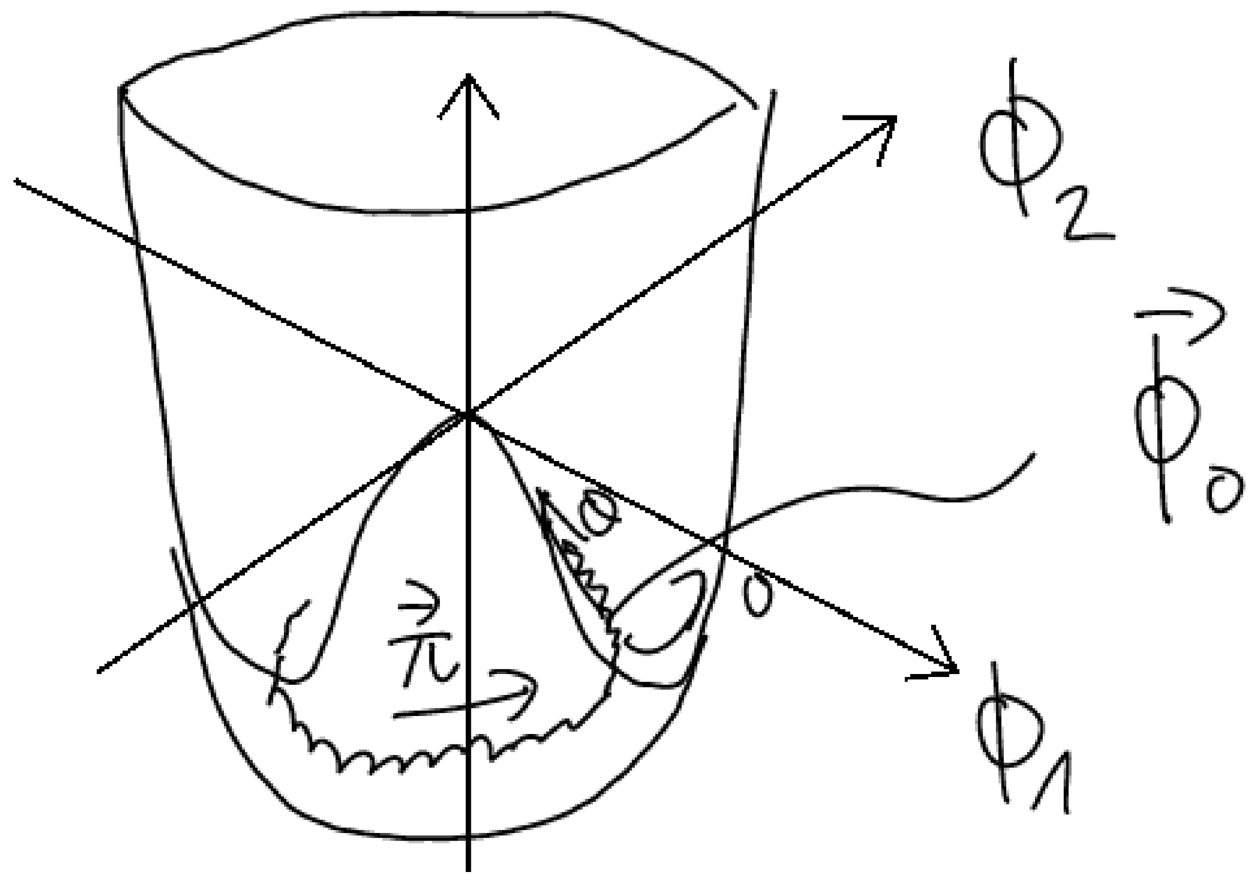
$R^T\cdot R = \mathbb{1} \;\hat{=}\; N\times N$ orthogonal matrix

the potential $V(\vec{\phi}) = -\tfrac{1}{2}\mu^2\vec{\phi}^T\vec{\phi} + \frac{\lambda}{4}(\vec{\phi}^T\vec{\phi})^2$

has a minimum at $\vec{\nabla}V = 0$

$$= -\mu^2\vec{\phi}_0 + \lambda(\vec{\phi}_0^T\vec{\phi}_0)\cdot\vec{\phi}_0 \rightsquigarrow \vec{\phi}_0^T\cdot\vec{\phi}_0 = \frac{\mu^2}{\lambda}$$

or $\boxed{|\phi_0|^2 = \frac{\mu^2}{\lambda}}$ $\hat{=}$ vector $\vec{\phi}_0$ with fixed length



mexican hat potential

with (hyper)-spherical coordinates

$$\vec{\phi}(x) = \begin{pmatrix} \frac{\mu}{\sqrt{\lambda}} + \vartheta(x) \\ \vec{\pi}(x) \end{pmatrix} \begin{matrix} \text{radius} \\ \text{angles} \end{matrix}$$

such that we hit the mima

for $\vartheta(x) = 0$ results in

$$\mathcal{L}(\vec{\phi}) = \frac{1}{2}\partial_\mu \vec{\pi}^T \partial^\mu \vec{\pi} - \frac{\lambda}{4}\left(\vec{\pi}^T\vec{\pi}\right)^2 + \qquad \text{I}$$

$$\frac{1}{2}\partial_\mu \vartheta \partial^\mu \vartheta - \frac{1}{2}(2\mu^2)\vartheta^2 - \sqrt{2}\mu\vartheta^3 - \frac{\lambda}{4}\vartheta^4 + \qquad \text{II}$$

$$-\sqrt{\lambda}\mu \vec{\pi}^T\vec{\pi}\vartheta - \frac{\lambda}{2}\vec{\pi}^T\vec{\pi}\vartheta^2 \qquad \text{III}$$

and therefore:

II ϑ is a massive $(m_\vartheta = \sqrt{2}\mu)$ scalar (like before)

I $\vec{\pi} = \pi^k$ with $k = 1, \dots, N-1$, massless fields with $O(N-1)$-symmetry

III interactions between $\vec{\pi}$ and ϑ

In general: Goldstone's theorem

> For every spontaneously broken, contineous symmetry, the theory contains a massless particle.

here: $O(N)$ breaks to $O(N-1)$

$$\left.\begin{aligned} \dim\, O(N) &= \frac{N(N-1)}{2} \\ \dim\, O(N-1) &= \frac{(N-1)(N-2)}{2} \end{aligned}\right\} \frac{(N-1)}{\cancel{2}}\left(\cancel{N}-\cancel{N}+\cancel{2}\right) = N-1 \text{ broken}$$

Symmetry generators

$\leadsto$ $N-1$ massless scalars $\hat{=}$ $\vec{\pi}$

9.2. Local symmetries & Higgsmechanism

remember SED from 7.5., now with potential

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi \overline{D^\mu \Phi} - V(\Phi))$$

we also have:

- covariant derivative $D_\mu = \partial_\mu + ieA_\mu$
- gauge transformations

$$\Phi'(x) = e^{i\Lambda(x)}\,\Phi \quad \text{and}$$
$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \Lambda(x)$$

again, we use the mexican hat potential

$$V(\Phi) = -\frac{\mu^2}{2}\Phi\overline{\Phi} + \frac{\lambda}{4}\left(\Phi\overline{\Phi}\right)^2$$ with a minimum at

$$|\Phi_0|^2 = \frac{\mu^2}{\lambda}$$ or in polar coordinates

$$\Phi(x) = \rho\, e^{i\Theta} \quad \text{for } \rho_0 \hat{=} \frac{\mu}{\sqrt{\lambda}} \quad \text{and}$$

$$V(\Phi) = V(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\lambda}{4}\rho^4$$ we get the full Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \rho \, \partial^\mu \rho + \frac{1}{2} \mu^2 \rho^2 - \frac{\lambda}{4} \rho^4$$
$$- \frac{1}{2} \rho^2 (\partial_\mu \theta + e A_\mu)(\partial^\mu \theta + e A^\mu)$$

a Poincaré invariant solution of the e.o.m.

is given by $\quad \rho = \frac{\mu}{\sqrt{\lambda}}, \quad \theta = \text{const}, \quad A_\mu = 0$

next week we expand around it.