reminder: open super strings have two sectors

Ramond ~ R
Neveu-Schwarz ~ NS } from ± in ferm. BC.

bosons and fermions in target space

tachyon is killed by GSO projection $(-1)^F = +1$
massless excitations furnish $\mathcal{N}=1$ gauge multiplet

for closed string left and right sector & level matching
R NS R NS

tachyon is in $(NS-, NS-)$ $(-1)^F$ eigenvalue
different GSO projections possible we are interested

in type IIA: $(NS+,NS+) \oplus (R+,NS+) \oplus (NS+,R-) \oplus (R+,R-)$
& type IIB: $-\text{"}- \oplus -\text{"}- \oplus (NS,R+) \oplus (R+,R+)$

fermions
bosons in the target space

NS-NS sector gives $1 \oplus 28 \oplus 35$ SO(8) little group irreps
dilaton ϕ B_{MN}-field metric

R-R sector

IIA $8_v \oplus 56 = \underset{C_{(1)}}{\square} \oplus \underset{C_{(3)}}{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline\end{array}}$ ~ traceless

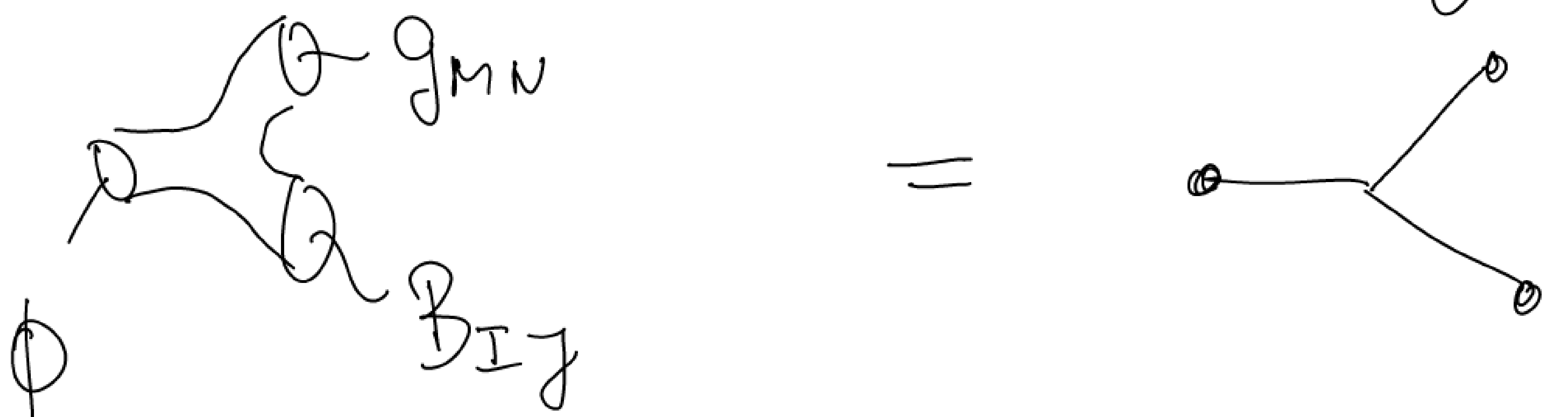
IIB $1 \oplus 28 \oplus 35_+ = \underset{C_{(0)}}{\bullet} \oplus \underset{C_{(2)}}{\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}} \oplus \underset{C_{(4)}}{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline\end{array}}$ self-dual

furnish $\mathcal{N}=(1,1)$ and $\mathcal{N}=(2,0)$ super gravity multiplet
typ IIA typ IIB

(SUGRA) in $D=10$.

8.5. Low energy effective actions

2^{nd} quantization of the string



from string perturbation theory (↗ 8.3)

from QFT for the low-energy effective action S

expansion in α' and g_s

Einstein frame

For type IIB in string frame, @ leading order

$$\int d^{10}x \sqrt{-g}\, e^{-\phi} R + \dots \quad \text{instead of} \quad \int d^{10}x \sqrt{-g}\, R + \dots$$

$$\boxed{\begin{aligned} S_{IIB} = \frac{1}{2\tilde{\kappa}_{10}^2}\Big[\int d^{10}x \sqrt{-g} \Big(& e^{-2\phi}\big(R + 4\partial_M \phi \partial^M \phi - 1/2 |H_{(3)}|^2\big) \\ & - \frac{1}{2}|F_{(1)}|^2 - \frac{1}{2}|\tilde{F}_{(3)}|^2 - \frac{1}{4}|\tilde{F}_{(5)}|^2 \Big) \\ & \underbrace{- \frac{1}{2}\int C_{(4)} \wedge H_{(3)} \wedge F_{(3)}}_{\text{topologic}} \Big] + \text{fermions} \end{aligned}}$$

with $|F_{(p)}|^2 = \frac{1}{p!} F_{N_1 \dots N_p} F^{N_1 \dots N_p}$,

the gravitational coupling $\boxed{2\kappa_{10}^2 = 2\tilde{\kappa}_{10}^2 g_s^2 = (2\pi)^7 \alpha'^4 g_s^2}$, and

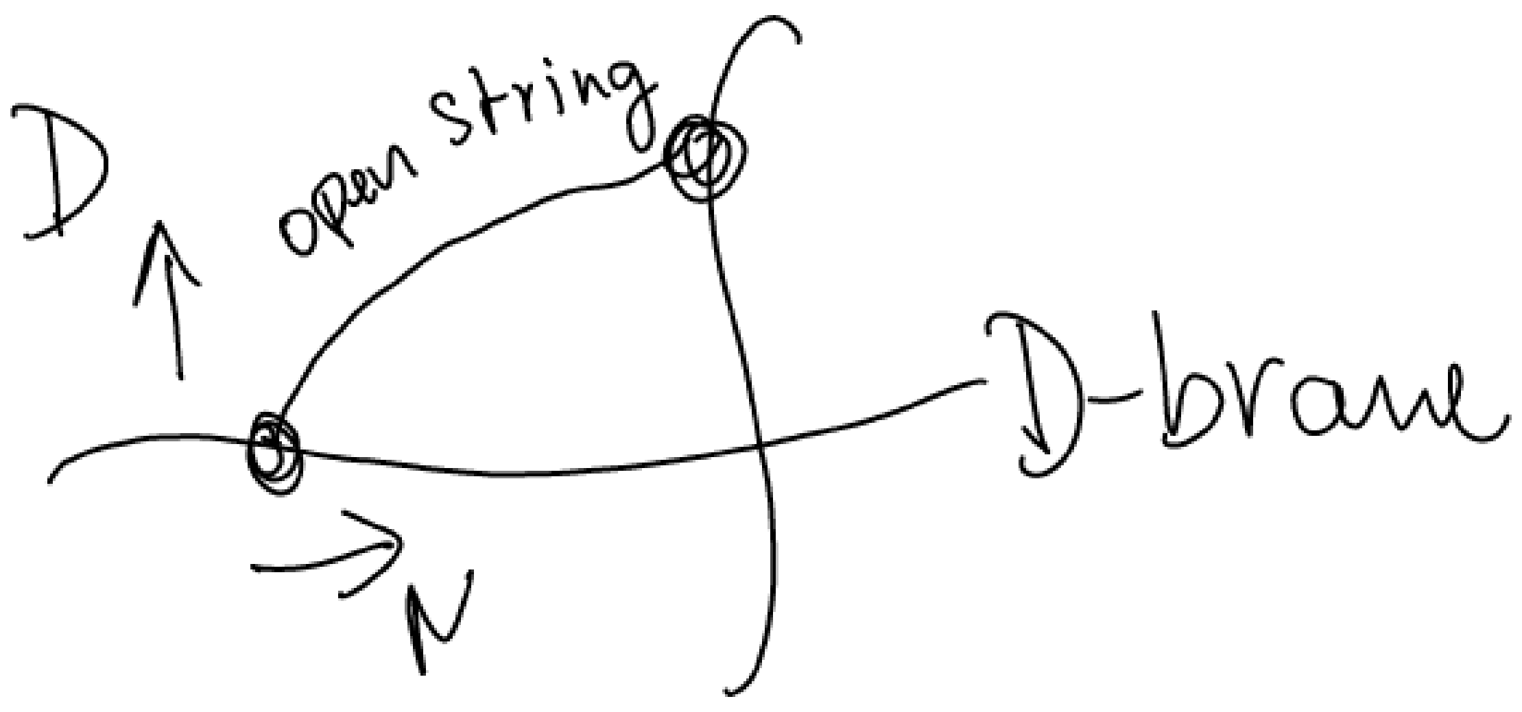
the p-form gauge fields $F_{(p)} = dC_{(p-1)}$, $H_{(3)} = dB_{(2)}$

$\tilde{F}_{(3)} = F_{(3)} - C_{(0)} H_{(3)}$, and $\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2} C_{(2)} \wedge H_3 + \frac{1}{2} B_{(2)} \wedge F_{(3)}$.

self-duality constraint $\boxed{* \tilde{F}_{(5)} = \tilde{F}_{(5)}}$

9. D-branes and non-perturbative objects

remember open string BCs

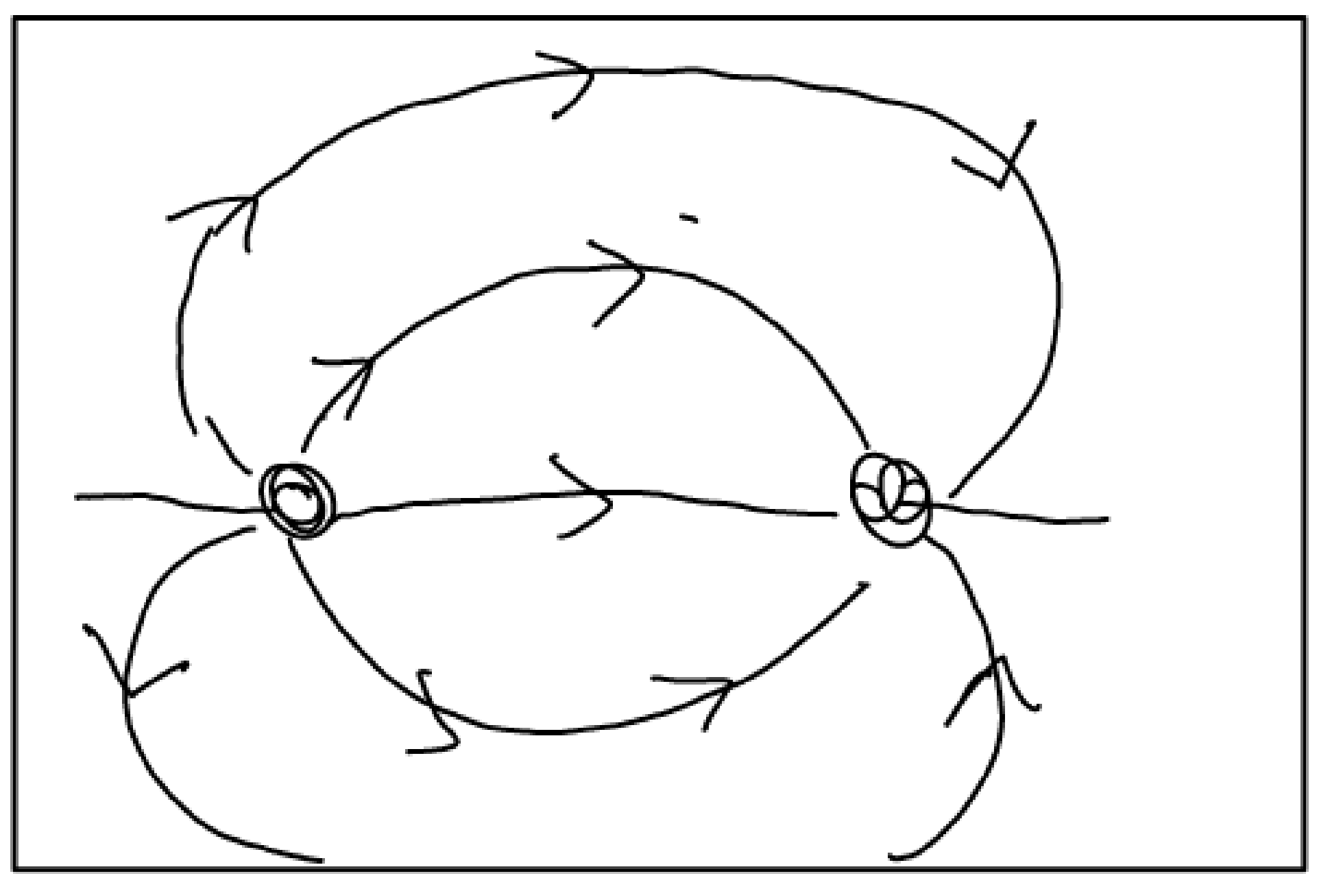


@ the worldsheet we get

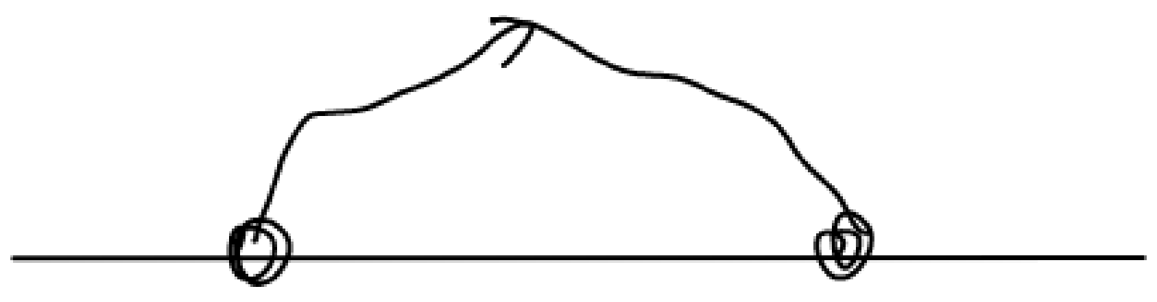
$$S = \int_\Sigma B + \dots \qquad \text{for } B = dA$$

$$= \int_{\partial\Sigma} A + \dots = \underbrace{\int d\tau \, A_M \partial_\tau X^M}\Big|_{\sigma=0}^{\sigma=\pi}$$

massless-excitations ≙ one-form gauge field



D-brane with charges
string endpoints on
world volume



low energy effective
action for the endpoints

$$\boxed{\underset{\text{Dirac-Born-Infield}}{S_{DBI}} = -T_p \int \overset{\text{Dp-brane}}{d^{p+1}\xi} \, e^{-\phi} \sqrt{-\det(P[g] + P[\mathcal{B}])}}$$

$$\overset{\text{pull back}}{(P[g])_{ab}} = \frac{\partial X^M}{\partial \xi^a} \frac{\partial X^N}{\partial \xi^b} g_{MN}, \quad \text{and} \quad \mathcal{B}_{MN} = B_{MN} + 2\pi\alpha' F_{MN}$$

$$F_{MN} = 2\,\partial_{[M} A_{N]}$$

using $\det(1+M) = 1 - \frac{1}{2}\mathrm{Tr}(M^2)$ for $M^T = -M$ we get

$$S_{DBI} = -(2\pi\alpha')^2 \frac{T_p}{4 g_s} \int d^{p+1}\xi \; F_{ab} F^{ab} + \dots$$

$$= U(1) \text{ YM-theory with } \boxed{g_{YM}^2 = \frac{g_s}{T_p (2\pi\alpha')^2} = (2\pi)^{p-2} g_s \alpha'^{\frac{p-3}{2}}}$$

⇝ **D-branes are dynamical**

Question: What happens to (R,R)-potentials?

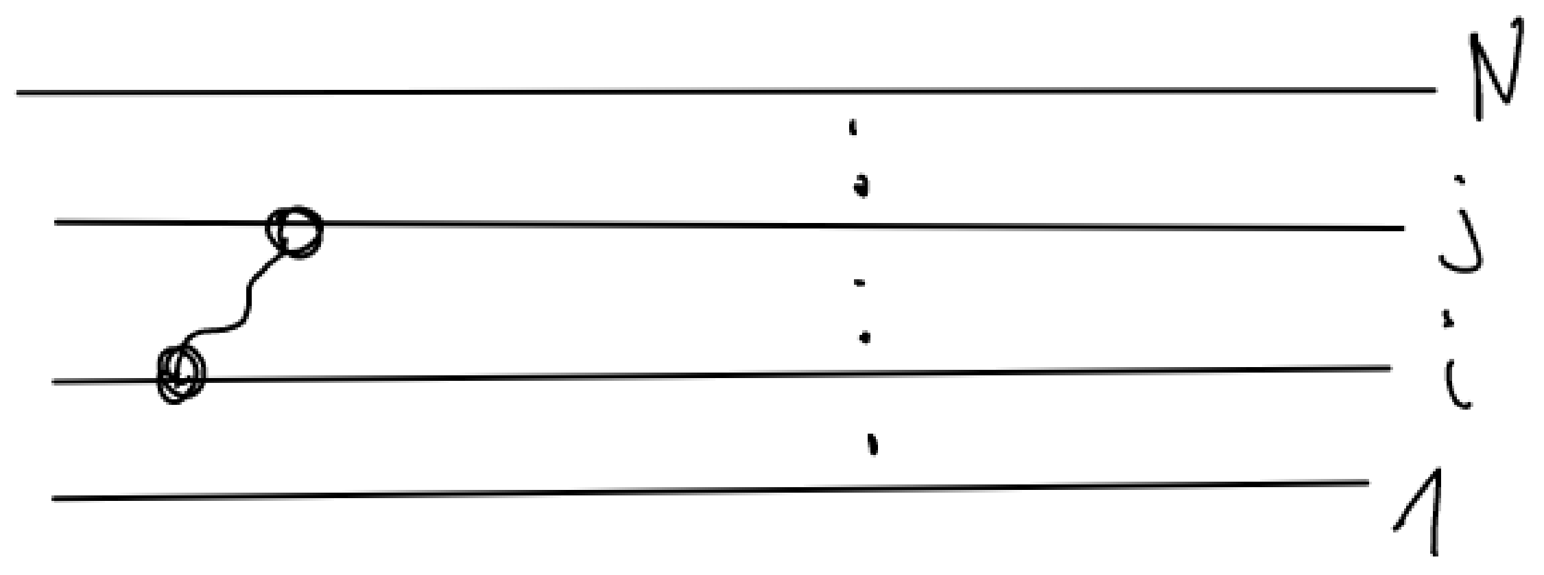
⇝ Chern-Simons term

$$\boxed{S_{CS} = \mu_p \int \sum_q P[C_{(p+1)}] \wedge e^{P[\mathcal{B}]}, \quad S = S_{DBI} + S_{CS}}$$

remarks: - $C_{(p+1)}$ is for D_p-brane what $B_{(2)}$ is for string

- D_{p-2n} branes can end on D_p branes

9.1. Stacks of D-branes



additional label λ_{ij} = Chan-Paton factor, global sym. on worldsheet

⇝ local in target space

for oriented string $\lambda \in U(N)$

- " un - " - $\lambda \in SO(N)$ or $USp(N)$

⇝ $\Omega: \sigma \to \pi - \sigma$ (O-plane)

⇝ U(N) YM-theory

$$C_{(p+1)} = \left(H_p(r)^{-1} - 1\right) dx^0 \wedge \ldots \wedge dx^p \,, \quad B_{MN} = 0$$

x^μ, $\mu = 0, \ldots, p$ coordinates on world volume

y^i, $i = p+1, \ldots, 9$ — " — ⊥ to — " — .

$r^2 = \sum_{i=p+1}^{9} (y^i)^2$ distance from brane

field equations imply $\boxed{\Box H_p(r) = 0}$

$$H_p(r) = \underbrace{1}_{\text{for } r \to \infty \text{ Minkowski space}} + \left(\frac{L_p}{r}\right)^{7-p}$$

with $\boxed{L_p^{7-p} = (4\pi)^{(5-p)/2}\,\Gamma\left(\frac{7-p}{2}\right) g_s N \alpha'^{(7-p)/2}}$