con't : canonical quantization of string

(1.) poisson brackets for $X_0^\mu, p^\mu, \alpha_n^\mu$ and $\tilde{\ }$ –n– ✓

(2.) $\{\cdot,\cdot\} \to -i[\cdot,\cdot]$

$$[x_0^\mu, p^\nu] = i\eta^{\mu\nu} \quad \text{and} \quad [\alpha_m^\mu, \alpha_n^\nu] = m\eta^{\mu\nu}\delta_{m,-n}$$

rescale $a_m^\mu = \frac{1}{\sqrt{m}}\alpha_m^\mu$ and $(a_m^\mu)^+ = \frac{1}{\sqrt{m}}\alpha_{-m}^\mu$

with $[a_m^\mu, (a_n^\nu)^+] = \eta^{\mu\nu}\delta_{mn}$ ~ harmonic oscillators

except for $[a_m^0, (a_m^0)^+] = -1$ ← negative norm states 

→ need to decouple

 Virasoro constraints help

⇝ solve them in light-cone coordinates

$$X^\pm = X^0 \pm X^{D-1} \quad \text{and gauge fixing} \quad i = 1, \dots, D-2$$

$$X^+ = x_0^+ + 2\alpha' p^+ \tau \leadsto X^-(X^+, X^i, x_0^-)$$

dynamical degrees of freedom are p^+, x_0^-, p^i, x_0^i and $a_m^i, (a_m^i)^+$

vacuum $p_0^\mu |0,k\rangle = k^\mu |0,k\rangle$ and therefore

$a_m^i |0,k\rangle = 0$

general state
$$|N,k\rangle = \prod_{i=1}^{D-2}\prod_{n=1}^{\infty} \frac{(a_n^{i+})^{N_{in}}}{\sqrt{N_{in}!}}|0,k\rangle$$

$$L_0 = \alpha' p^\mu p_\mu + \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} n\, a_n^{i+} a_n^i = \alpha' p^\mu p_\mu + N$$

Virasoro constraints require on physical state $|\psi\rangle$

$(L_0 - a)|\psi\rangle = 0$ and $L_n|\psi\rangle = 0$ for all $n \in \mathbb{N}$

ordering ambiguity $a = \frac{D-2}{2} \sum_{n \geq 0} n = -\frac{D-2}{24}$

ζ-function regularization

$$\boxed{M^2 = -k^\mu k_\mu = \frac{1}{\alpha'}\left(N + \frac{2-D}{24}\right)} \rightsquigarrow \text{massless-state}$$

$(a_1^i)^\dagger |0, k\rangle$ with $N = 1$ and $M^2 = \frac{D-26}{24} \overset{!}{=} 0$

transforms in little group $SO(D-2)$, like photon

for open string because $\tilde{\alpha}_m^N$ gets removed by b.c.

$\lightning$ for $N = 0$ $M^2 < 0$ $\rightsquigarrow$ tachyon removed by SUSY later

$$\boxed{D = 26 \text{ is the critical dim. for bos. string}}$$

for closed string a_m^i and $\tilde{a}_m^i$, but level matched

$N = \tilde{N}$ with massless state

$a_1^i \tilde{a}_1^j |0, 0, k\rangle$, $M^2 = \frac{26-D}{6\alpha'} = 0$

$\rightsquigarrow$ decomposes into $\square \times \square = \begin{smallmatrix}\square\\\square\end{smallmatrix} + \square\square + \bullet$

Kalb-Ramon field (B) graviton dilaton

Conclusion: • open string $\rightarrow$ gauge theory

• closed strings $\longrightarrow$ gravity

8.3. String perturbation theory

remember ST $\hat{=}$ 2d gravity

Euler characteristic

$$S_P' = S_P - \lambda \chi \qquad \chi = \frac{1}{4\pi}\int_\Sigma d^2\sigma \sqrt{h}\, R$$

only matter until now

Einstein-Hilbert

with generating function

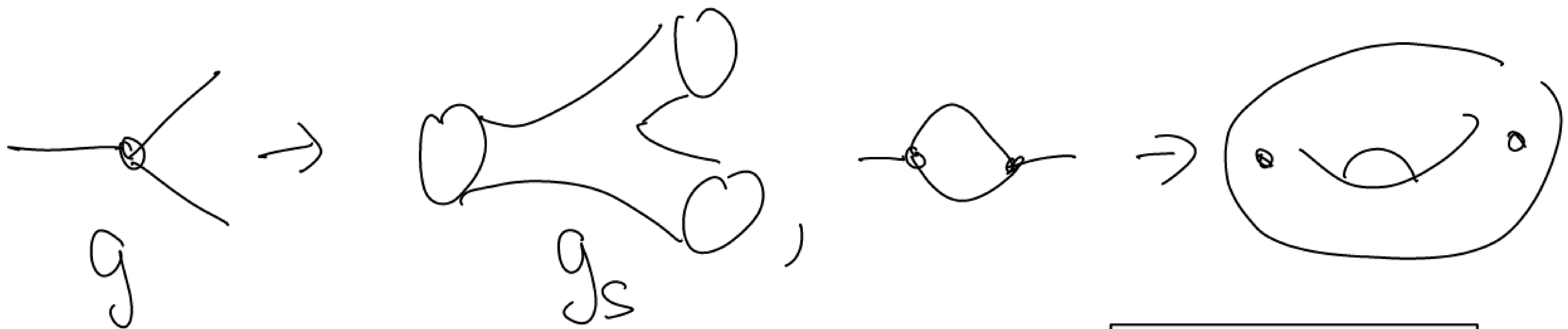
because $\chi = 2-2g$

$$Z = \int_\Sigma \mathcal{D}X^M \mathcal{D}h_{\alpha\beta}\, e^{-S_P'} = \sum_{g=0}^{\infty} \int_{\Sigma_g} \mathcal{D}X^M \mathcal{D}h_{\alpha\beta}\, e^{-S_P'}$$

$$= \sum_{g=0}^{\infty} \underbrace{e^{-\lambda}}_{1/g_s}{}^{(2-2g)} \int_{\Sigma_g} \mathcal{D}X^M \mathcal{D}h_{\alpha\beta}\, e^{-S_P}$$

$1/g_s$ string coupling

In a picture



comparing with large N, we identify $\boxed{1/g_s = N}$

8.4. Superstrings

$$\boxed{S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\, \eta^{\alpha\beta}\left(\partial_\alpha X^M \partial_\beta X^N + i\bar{\Psi}^M \gamma_\alpha \partial_\beta \Psi^N\right) g_{MN}}$$

$\Psi^M = (\Psi^M_-, \Psi^M_+)^T$ M Majorana spinors (two real components)

with SUSY variations $\delta_\varepsilon X^M = \bar{\varepsilon}\Psi^M$ & $\delta_\varepsilon \Psi^M = \gamma^\alpha \partial_\alpha X^M \varepsilon$

field eq.: bosonic part + $\partial_+ \Psi_-^M = \partial_- \Psi_+^M = 0$ and

and for the open string

$$\underset{\text{fermionic part}}{\delta S_f} = \frac{i}{4\pi\alpha'} \int d\tau \left(\Psi_-^M \delta \Psi_{-M} - \Psi_+^M \delta \Psi_{+M} \right) \Big|_{\sigma=0}^{\sigma=\pi} = 0$$

because of i.b.p.

b.c. are therefore

Ramond — R: $\Psi_+^M(\tau,\pi) = +\Psi_-^M(\tau,\pi)$ or

Neveu-Schwarz — NS: $\Psi_+^M(\tau,\pi) = -\Psi_-^M(\tau,\pi)$

with the mode expansions

$$R: \Psi_\mp^M(\tau,\sigma) = \frac{1}{\sqrt{2}} \sum_{n\in\mathbb{Z}} d_n^M e^{-in\sigma_\mp}$$

$$NS: \Psi_\mp^M(\tau,\sigma) = \frac{1}{\sqrt{2}} \sum_{r\in\mathbb{Z}-1/2} b_r^M e^{-ir\sigma_\mp}$$

again modes to operators with anti-commutators

$$\{d_m^M, d_n^N\} = \eta^{MN} \delta_{m,-n} \quad \& \quad \{b_r^M, b_s^N\} = \eta^{MN} \delta_{r,-s}$$

Vacuum: $b_r^M |0\rangle_{NS} = 0$ for $r > 0$ and (scalar)

$d_r^M |0\rangle_R = 0$ — " — (spinor)

degenerated because

$$\{d_m^M, d_0^N\} = 0$$ and therefore

$d_0^N |0\rangle_R$ is another ground state

Γ^N $\rightsquigarrow$ $|0\rangle_R$ is space time spinor

massless states: $b_{-1/2}^i |0\rangle_{NS}$ (like bos. string)

$$M^2 = \frac{1}{\alpha'}\left(\frac{1}{2} - \frac{D-2}{16}\right) = 0 \rightsquigarrow \boxed{D=10}$$

lightest string states

GSO projection (Gliozzi, Scherk, Olive)

sector	$(-1)^F$	$SO(8)$	m^2
NS	$+1$	8_v	0
~~NS~~	~~-1~~	~~1~~	~~$-1/2\alpha'$~~
R	$+1$	8	0
~~R~~	~~-1~~	~~$8'$~~	~~0~~

chiral spinor

anti – u

$\rightsquigarrow$ massless states $\{\underbrace{b^i_{-1/2}|0\rangle_{NS}}_{\text{gauge boson}}, \underbrace{|0\rangle_R}_{\text{gaugino}}\}$

$\mathcal{N}=1$ gauge multiplet