An Algebraic Classification of Solution Generating Techniques

joint work with R. Borsato and S. Driezen

1. Motivation

find solutions to closed string's low energy effective action:

$$S=\int d^Dx\sqrt{g}\,e^{-2\phi}\left(R+4(\partial_i\phi)^2-\frac{1}{12}H_{ijk}H^{ijk}\right)$$
$$+\ \text{R/R field}\ +\ \text{higher deriv.}$$

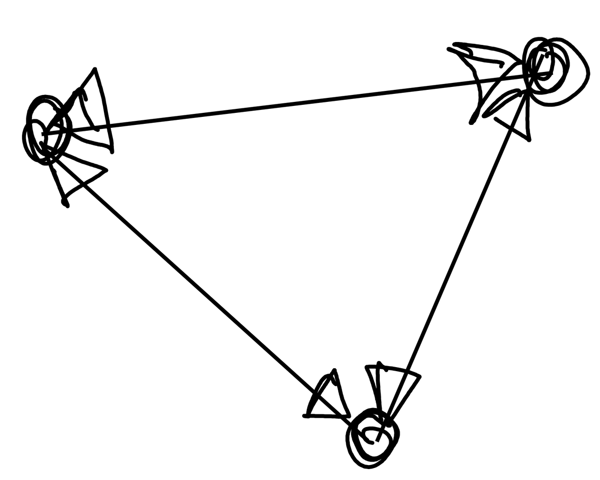
important but hard problem

- AdS/CFT
- Pheno ...

- solve highly coupled PDE's

> Use one known solution as a "seed" for new solutions

Solution "Landscape"



~ duality

Δ plurality

→ very restrictive!

Examples:
- abelian T-duality
- S-duality

Problem: require abelian isometries

Question: • are there more less restrictive dualities?

YES! Poisson-Lie T-duality

2. Making Hidden Structures Manifest

unify metric g_{ij} and B-field B_{ij} in

gen. frame-field

$$\mathcal{H}_{IJ}=\left(\begin{array}{c|c} g^{ij} & -B_{ik}g^{kj} \\ \hline g^{ik}B_{kj} & g_{ij}-B_{ik}g^{kl}B_{lj}\end{array}\right)=E^A{}_I\,\mathcal{H}_{AB}\,E^B{}_J$$

gen. metric

constant

and dilaton ϕ and metric in $d = \phi - \frac{1}{2}\log\sqrt{g}$

introduce gen. flux tensors $\sim$ H-flux $H = dB$

$$F_{ABC} = 3 E_{[A}{}^{I} \partial_I E_B{}^J E_{C]J}$$

$$\partial_I = \begin{pmatrix} 0 & \partial^i \end{pmatrix}, \quad \underbrace{\eta_{IJ} = \begin{pmatrix} 0 & \delta_j^i \\ \delta^j_i & 0 \end{pmatrix}, \quad \eta_{IK}\eta^{KJ} = \delta_I^J}_{\text{to raise \& lower indices}}$$

$$F_A = 2 E_A{}^I \partial_I d - \partial_I E_A{}^I$$

$\hookrightarrow$
$$S = \int d^Dx \Big[\mathcal{H}^{AB}(2 D_A F_B - F_A F_B) + F_{ABC} F_{DEF} \Big(\frac{1}{4} \mathcal{H}^{AD} \eta^{BE} \eta^{CF} - \frac{1}{12} \mathcal{H}^{AD} \mathcal{H}^{BE} \mathcal{H}^{CF} \Big) \Big]$$

$$D_A := E_A{}^I \partial_I$$

similar for field eqs.

3. Generalised Dualities

Duality (or SGT) leaves D_A, F_{ABC}, F_A, η_{AB}, $\mathcal{H}_{AB}$ invariant but changes $E_A{}^I$.

here special case: F_{ABC} & F_A are <u>constant</u>

$\hookrightarrow$ BI imply structure coeff. of 2D-dim. Lie algebra

Ingredients for $E_A{}^I$

1. Lie algebra $[T_A, T_B] = F_{AB}{}^C T_C \quad T_A \in \mathrm{Lie}(\mathbb{D})$
2. ad-invariant non-deg. pairing
   $$\langle T_A, T_B \rangle = \eta_{AB}$$
3. maximally isotropic subgroup $\mathbb{H}$

$T^a \in \mathrm{Lie}(\mathbb{H})$, $a = 1, \dots, D$, $\langle T^a, T^b \rangle = 0$

4. element from the center $F := F^A T_A$ with $[F, T_A] = 0$ ($F = 0$ always works)

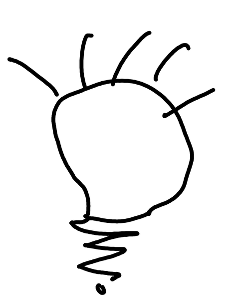
construct $E_A{}^I$ and d on coset $\mathbb{H} \backslash \mathbb{D}$

$$\mathbb{D}, F, \mathcal{H}, \eta$$

$$\boxed{\mathbb{H}_1 \backslash \mathbb{D}, \phi^{(1)}, g^{(1)}_{ij}, B^{(1)}_{ij}} \quad \dots \quad \boxed{\mathbb{H}_n \backslash \mathbb{D}, \phi^{(n)}, g^{(n)}_{ij}, B^{(n)}_{ij}}$$

4. Classification

Question: How many max. iso. subgroups for given $\mathbb{D}$.

 Take one $\mathbb{H}$ and deform it.

Deformations governed by (Lie algebra) cohomology

dual "one-form"

$$T^a \lrcorner\, \Theta_b = \delta^a_b \qquad d\Theta_a = -\frac{1}{2} F_a{}^{bc}\, \Theta_b \wedge \Theta_c$$

gen. of $\mathbb{H}$

$$d^2 \cdot = 0$$

$$H^n(\mathrm{Lie}(\mathbb{H}), \mathbb{R}) = \frac{\text{closed}}{\text{exact}} \text{ n-forms} \qquad \begin{array}{l} d\omega = 0 \\ \omega = d\lambda \end{array}$$

non-triv. inf. def. of $\mathbb{H}$ in $\mathbb{D}$ are classified by $\underline{H^2(\mathrm{Lie}(\mathbb{H}), \mathbb{R})}$ obstructions to make them finite are in $\underline{H^3(\mathrm{Lie}(\mathbb{H}), \mathbb{R})}$

5. Outlook

- even more general dualities, i.e. dressing cosets
- higher derivative (quantum)-corrections
- applications to AdS/CFT or pheno