Exploring Stringy Geometries with Double Field Theory

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in collaboration with

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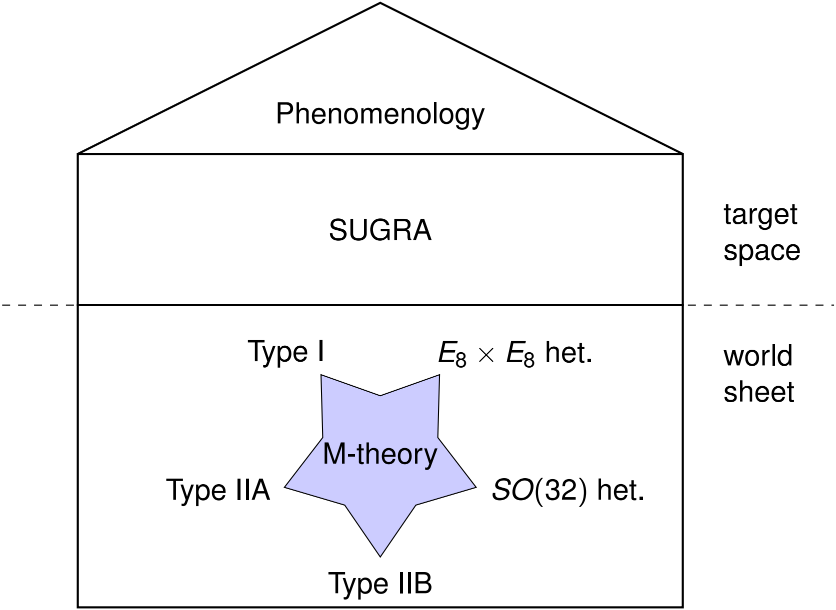
The big picture



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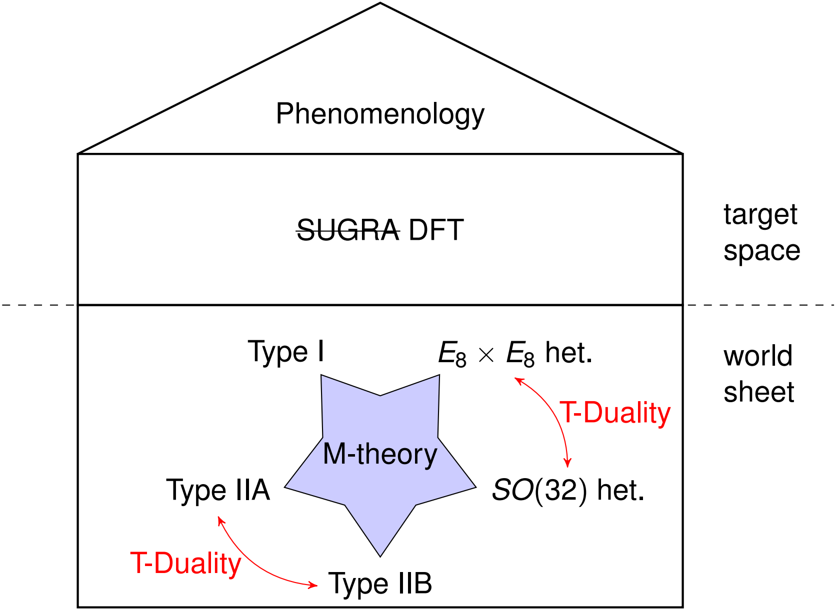

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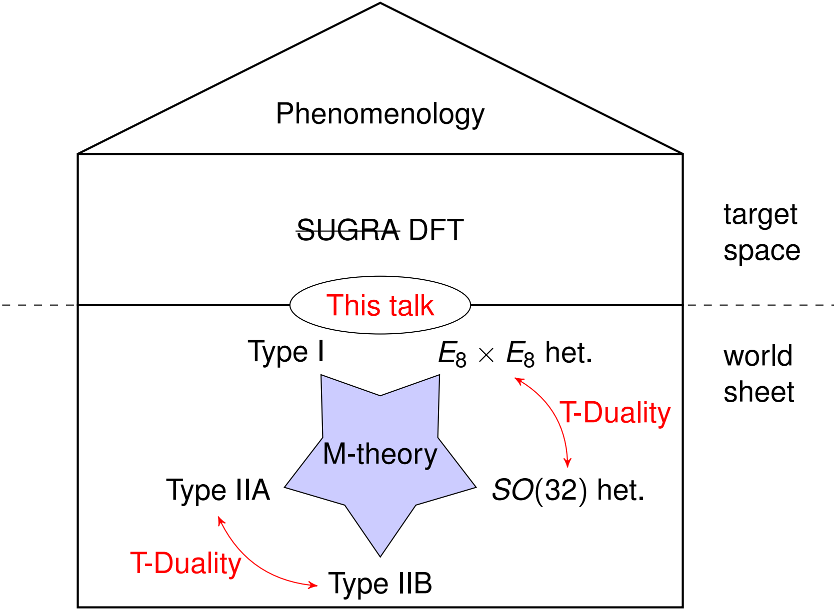


Outline

1. Double Field Theory in a nutshell

2. What is it good for? Current Problems? Solution?

3. Deriving DFT_{WZW} from CSFT

4. Generalized metric formulation

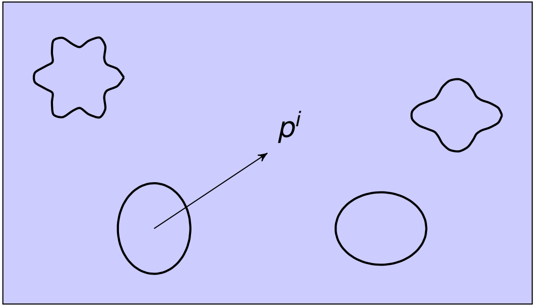
5. Generalized Scherk-Schwarz compactifications

6. Summary and outlook

SUGRA

- closed strings in D-dim. flat space with momentum p^i
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\mathrm{NS}} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4\partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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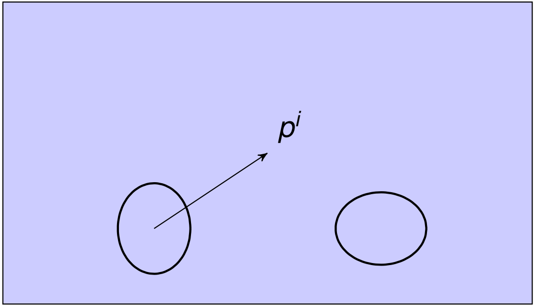

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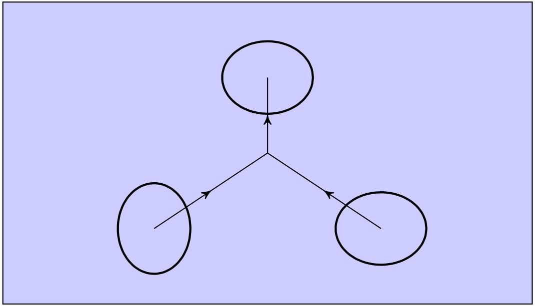
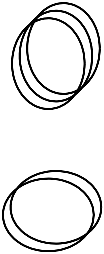




$g_{ij}(p^k)$

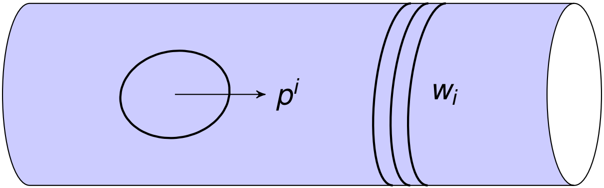
$\phi(p^i)$

DFT (Double Field Theory) [Siegel, 1993,Hull and Zwiebach, 2009,Hohm, Hull, and Zwiebach, 2010]

- closed strings on a flat torus with momentum p^i and winding w_i
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix}$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



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$$+\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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- lower/raise indices with $\eta_{MN} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^i_j & 0 \end{pmatrix}$ and $\eta^{MN} = \begin{pmatrix} 0 & \delta^i_j \\ \delta_i^j & 0 \end{pmatrix}$

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Gauge transformations [Hull and Zwiebach, 2009]

- generalized Lie derivative combines

  1. diffeomorphisms
  2. B-field gauge transformations
  3. β-field gauge transformations

  (1. and 2.: available in SUGRA)

$$\mathcal{L}_\lambda \mathcal{H}^{MN} = \lambda^P \partial_P \mathcal{H}^{MN} + (\partial^M \lambda_P - \partial_P \lambda^M)\mathcal{H}^{PN} + (\partial^N \lambda_P - \partial_P \lambda^N)\mathcal{H}^{MP}$$

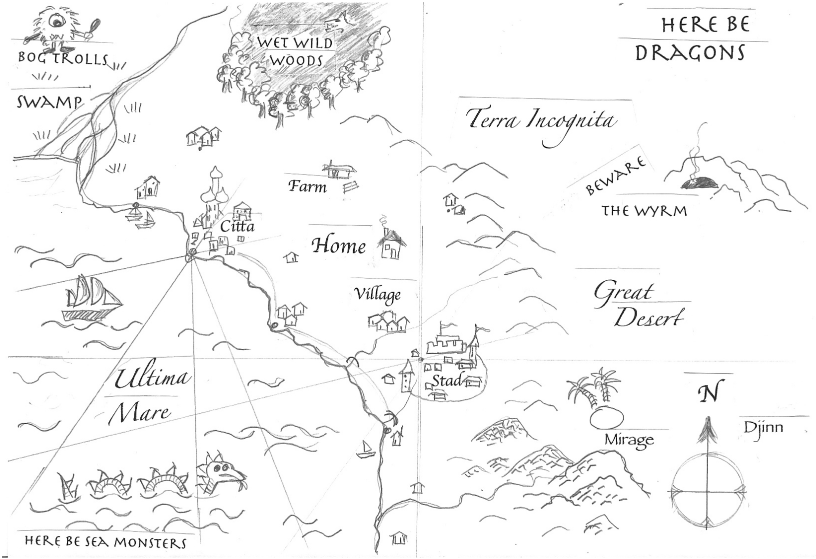
$$\mathcal{L}_\lambda d = \lambda^M \partial_M d + \frac{1}{2}\partial_M \lambda^M$$

- closure of algebra

$$\mathcal{L}_{\lambda_1}\mathcal{L}_{\lambda_2} - \mathcal{L}_{\lambda_2}\mathcal{L}_{\lambda_1} = \mathcal{L}_{\lambda_{12}} \quad \text{with} \quad \lambda_{12} = [\lambda_1, \lambda_2]_{\mathrm{C}}$$

- only if strong constraint holds

A landscape of string backgrounds [Douglas, 2003,Susskind, 2003]



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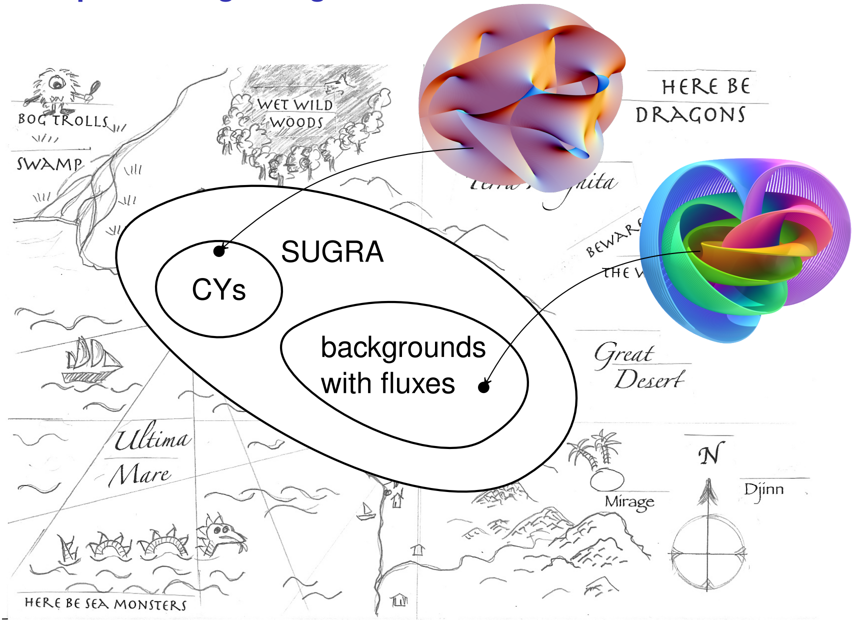

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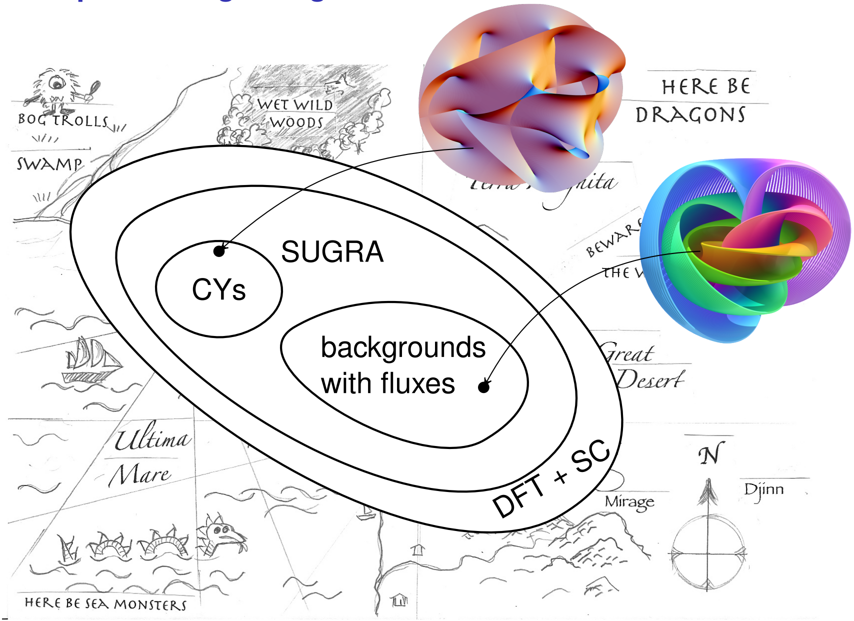

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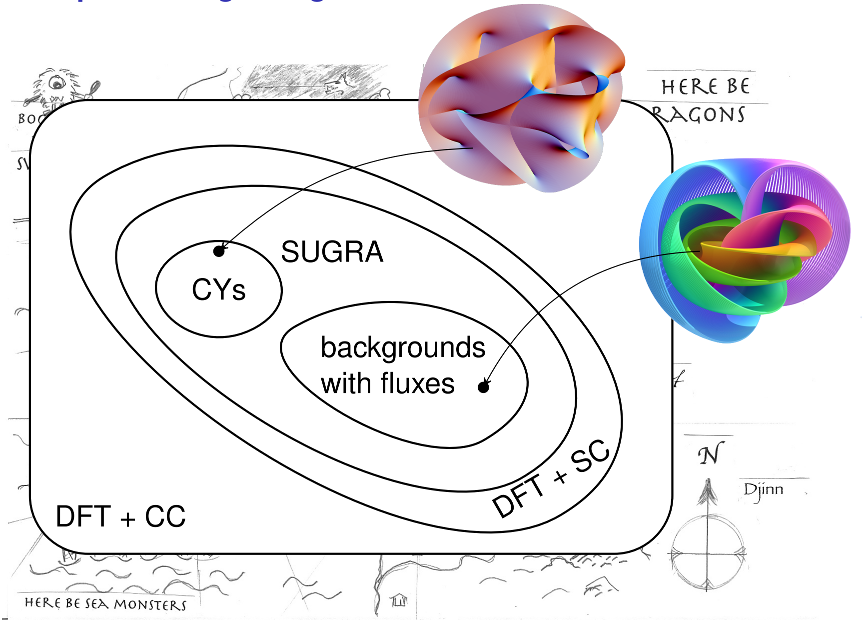

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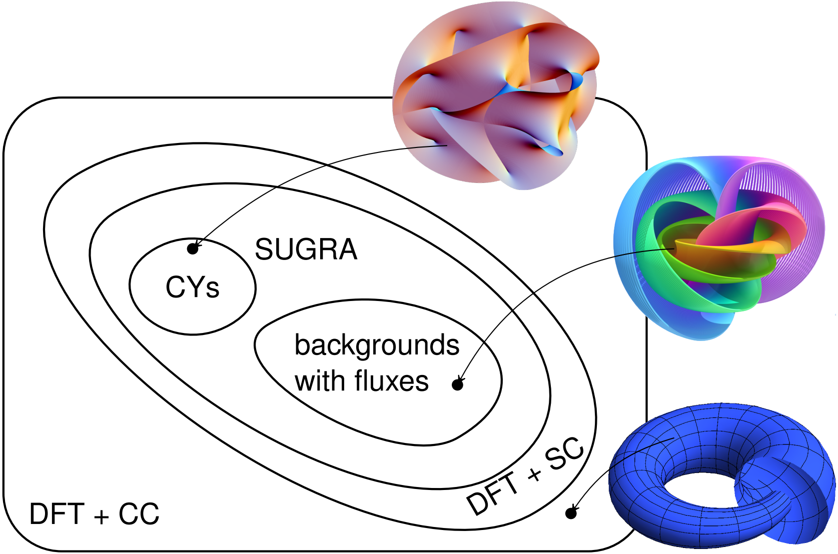

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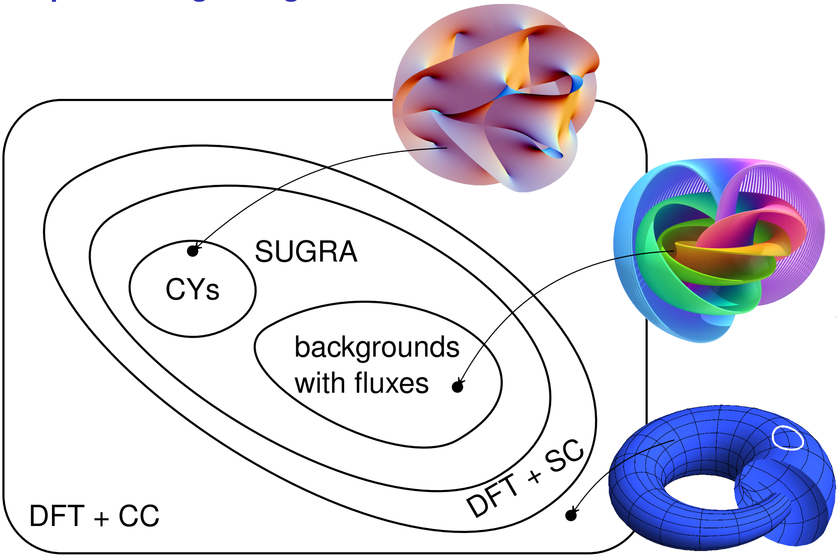

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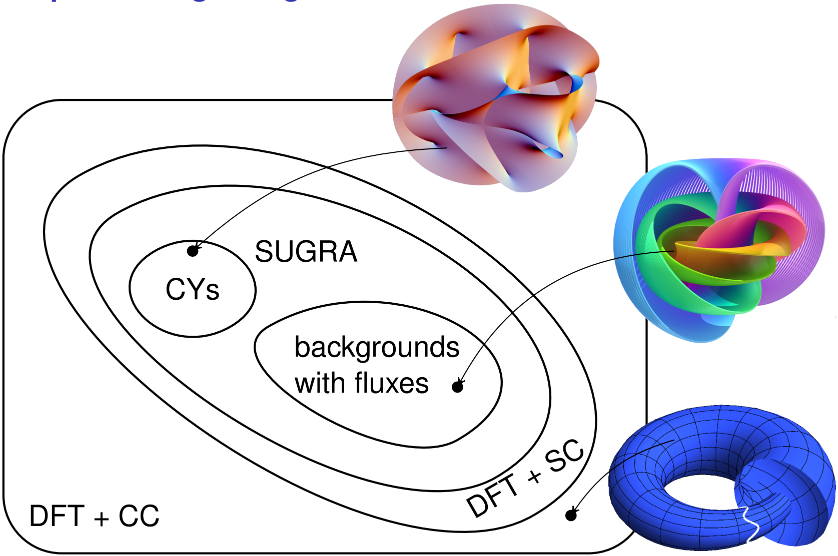

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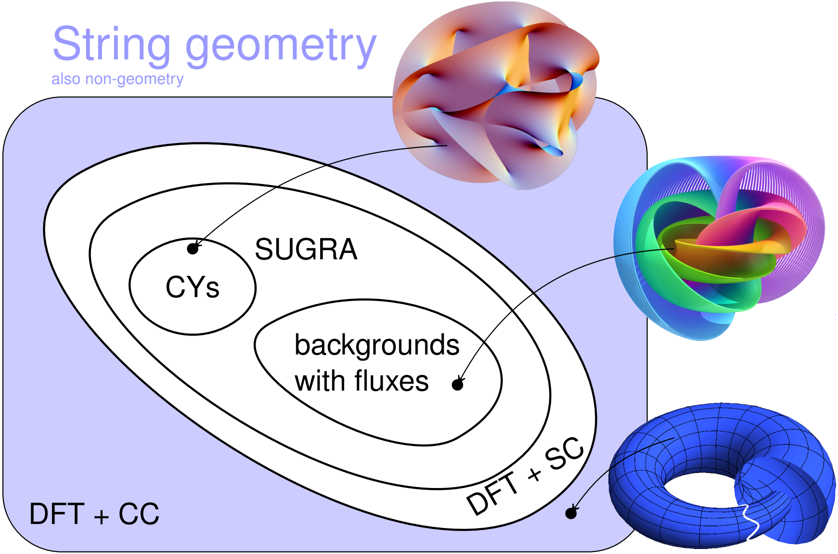

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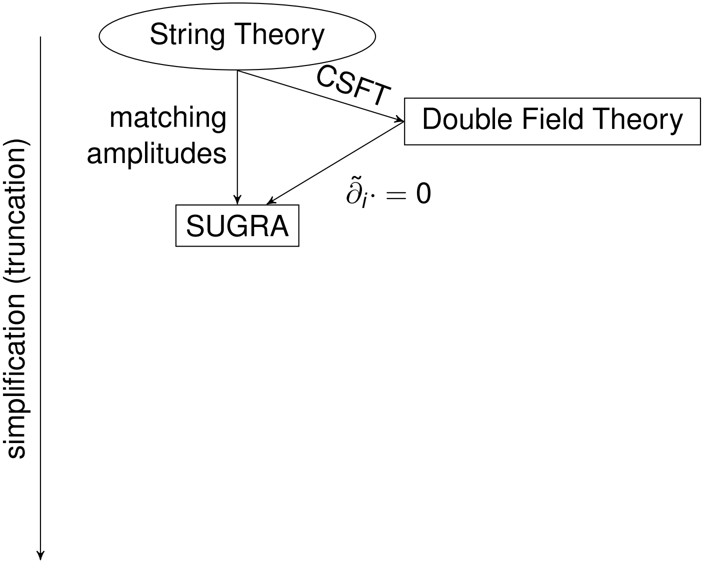
Generalized Scherk-Schwarz compactification

[Aldazabal, Baron, Marques, and Nunez, 2011,Geissbuhler, 2011]



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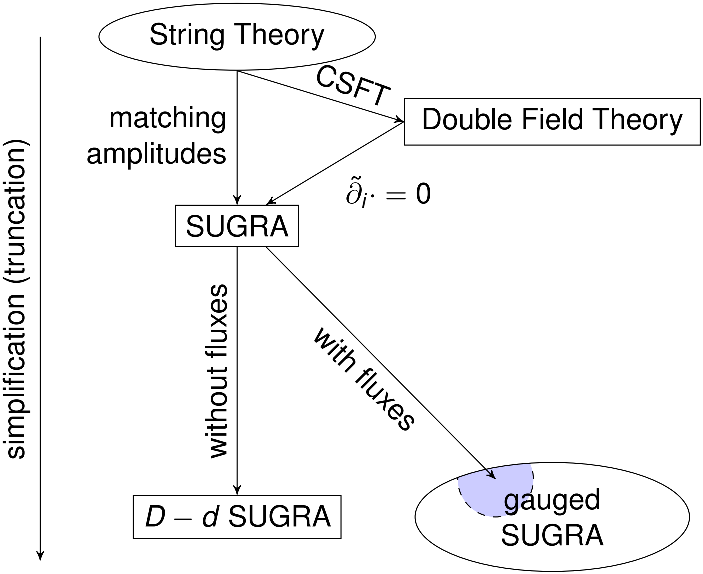

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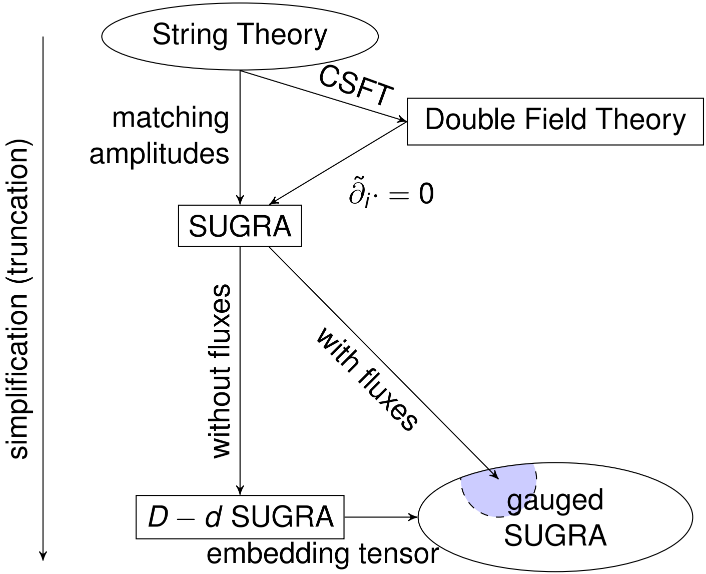

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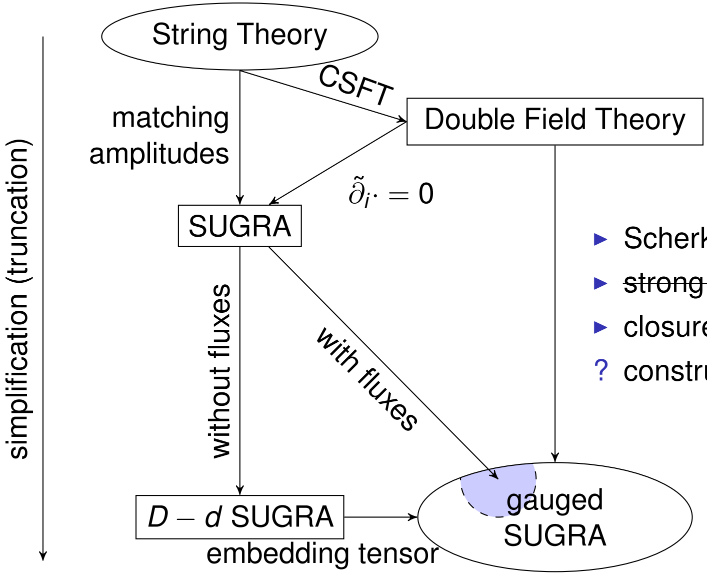

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- ~~strong constraint~~
- closure constraint
- ? construction twist

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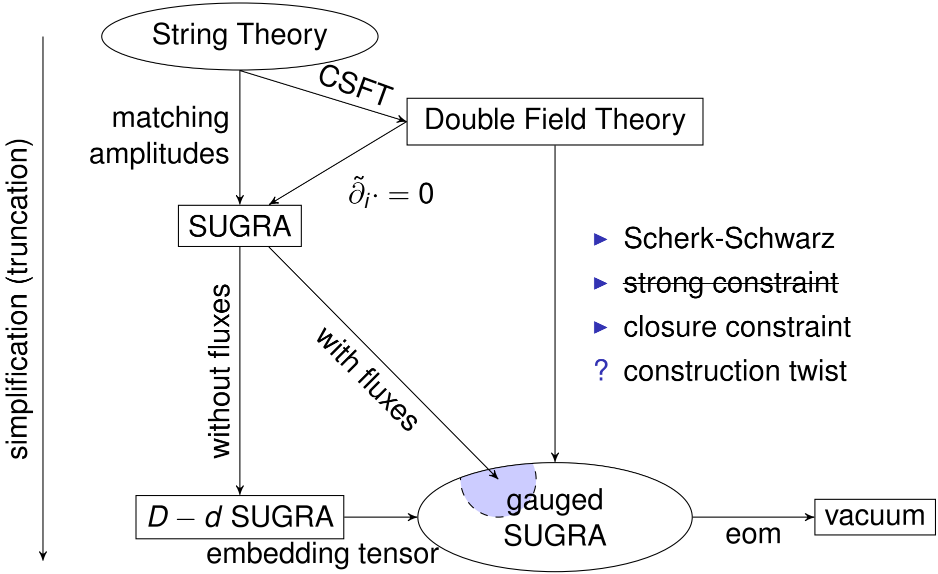

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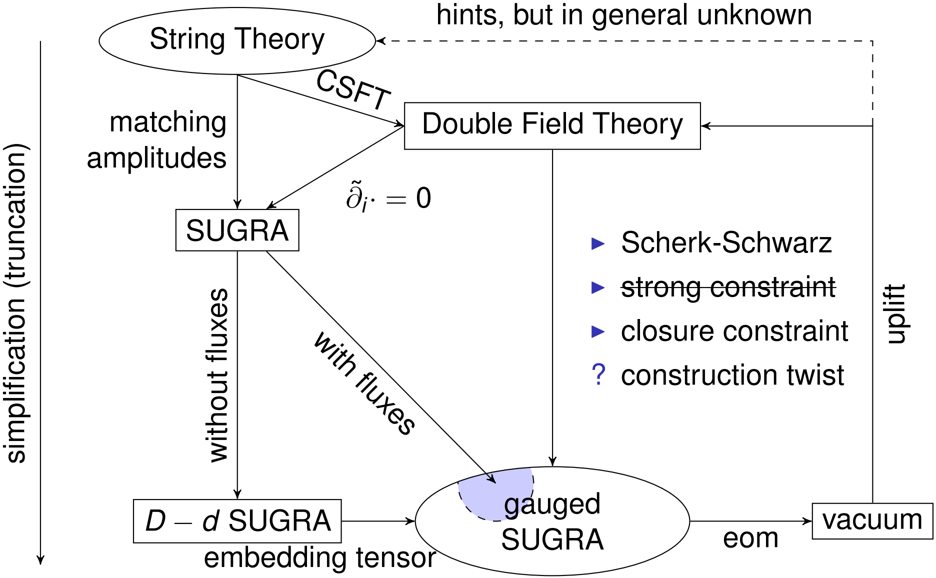

S^3, the elephant in the room

- switch on H-flux, solve eom

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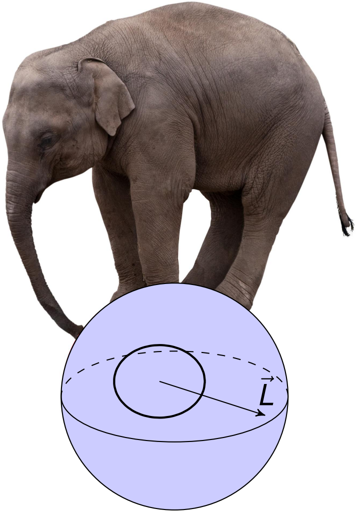

- switch on H-flux, solve eom
- no T^3 but S^3

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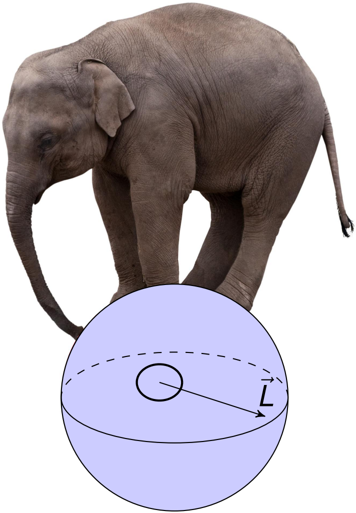

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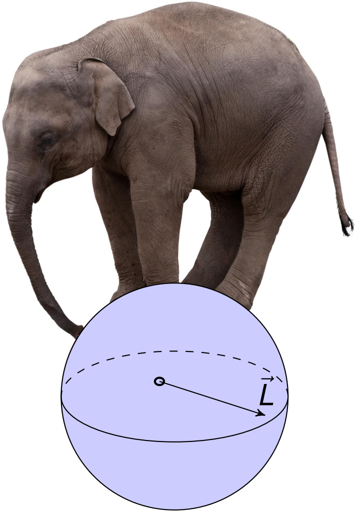

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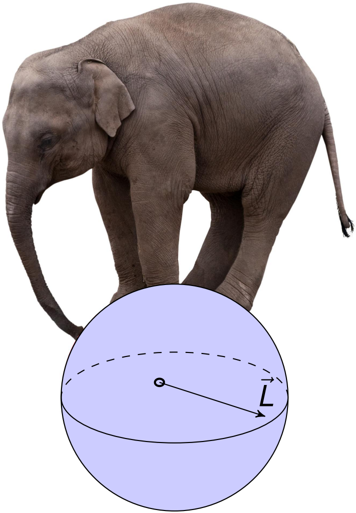

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- NEW! action
- NEW! symmetries

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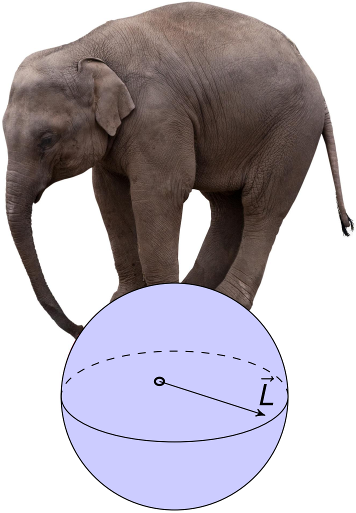

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FREE twist gen. Scherk-Schwarz

FREE genuinely non-geometric backgr.

DFT_{WZW} = DFT on group manifolds

Use group manifold instead of a torus to derive DFT!

+ includes $\begin{cases} \mathrm{T}^D = \mathrm{U}(1)^D \\ S^3 = \mathrm{SU}(2) \end{cases}$
+ CFT exactly solvable
+ flux backgrounds with const. fluxes

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- treat left and right mover independently
- $2D$ independent coordinates

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$$x^i = \frac{1}{\sqrt{2}}(x^i_{\mathrm{L}} + x^i_{\mathrm{R}})$$
$$\tilde{x}_i = \frac{1}{\sqrt{2}}(x_{\mathrm{L}i} - x_{\mathrm{R}i})$$

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Questions about DFT_{WZW}

- What are the covariant objects?
- How is it connected to DFT?
- Does it make non-abelian duality manifest?

} not trivial

WZW model & Kač-Moody algebra [Witten, 1983,Walton, 1999]

- $g \in G$, a compact simply connected Lie group

$$S_{\mathrm{WZW}} = \frac{1}{2\pi\alpha'} \int_M d^2z \mathcal{K}(g^{-1}\partial g, g^{-1}\bar{\partial} g) + S_{\mathrm{WZ}}(g)$$

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- metric and 3-form flux in flat indices

$$\eta_{ab} := \mathcal{K}(t_a, t_b) \quad \text{and} \quad F_{abc} := \mathcal{K}([t_a, t_b], t_c)$$

- D chiral and D anti-chiral Noether currents (=2D indep. currents)

$$j_a(z) = \frac{2}{\alpha'}\mathcal{K}(\partial g g^{-1}, t_a) \quad \text{and} \quad j_{\bar{a}}(\bar{z}) = -\frac{2}{\alpha'}\mathcal{K}(g^{-1}\bar{\partial} g, t_{\bar{a}})$$

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- radial quantization

$$\boxed{j_a(z) j_b(w) = -\frac{\alpha'}{2}\frac{1}{(z-w)^2}\eta_{ab} + \frac{1}{z-w}F_{ab}{}^c j_c(z) + \dots}$$

Action

- tree level action in CSFT [Zwiebach, 1993]

$$(2\kappa^2)S = \frac{2}{\alpha'}\left(\langle\Psi|c_0^- Q|\Psi\rangle + \frac{1}{3}\{\Psi,\Psi,\Psi\}_0 + \dots\right)$$

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- string field for massless excitations [Hull and Zwiebach, 2009]

$$|\Psi\rangle = \sum_R \Big[\frac{\alpha'}{4}\epsilon^{a\bar{b}}(R) j_{a\,-1} j_{\bar{b}\,-1} c_1 \bar{c}_1 + e(R) c_1 c_{-1} + \bar{e}(R)\bar{c}_1 \bar{c}_{-1} + \frac{\alpha'}{2}\left(f^a(R) c_0^+ c_1 j_{a\,-1} + f^{\bar{b}}(R) c_0^+ \bar{c}_1 j_{\bar{b}\,-1}\right)\Big]|\phi_R\rangle$$

- R is highest weight of $\mathfrak{g} \times \mathfrak{g}$ representation

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$$(2\kappa^2)S = \frac{2}{\alpha'}\left(\langle\Psi|c_0^- Q|\Psi\rangle + \frac{1}{3}\{\Psi,\Psi,\Psi\}_0 + \dots\right)$$

- string field for massless excitations [Hull and Zwiebach, 2009]

$$|\Psi\rangle = \sum_R \Big[\frac{\alpha'}{4}\epsilon^{a\bar{b}}(R) j_{a\,-1} j_{\bar{b}\,-1} c_1 \bar{c}_1 + e(R) c_1 c_{-1} + \bar{e}(R)\bar{c}_1 \bar{c}_{-1} + \frac{\alpha'}{2}\left(f^a(R) c_0^+ c_1 j_{a\,-1} + f^{\bar{b}}(R) c_0^+ \bar{c}_1 j_{\bar{b}\,-1}\right)\Big]|\phi_R\rangle$$

- R is highest weight of $\mathfrak{g}\times\mathfrak{g}$ representation
- BRST operator (L_m from Sugawara construction)

$$Q = \sum_m \left(: c_{-m} L_m : + \frac{1}{2} : c_{-m} L_m^{gh} :\right) + \text{anti-chiral}$$

Geometric representation of primary fields ($k \to \infty$)

- flat derivative $\boxed{D_a = e_a{}^i \partial_i \quad \text{with} \quad e_a{}^i = \mathcal{K}(g^{-1}\partial^i g, t_a)}$

operator algebra	geometry ($j_{a\,0} \to D_a$)
$L_0\lvert\phi_R\rangle = j_{a\,0} j^a_0 \lvert\phi_R\rangle = h_R\lvert\phi_R\rangle$	$D_a D^a Y_R(x^i) = h_R Y_R(x^i)$
$[j_{a\,0}, j_{b\,0}] = F_{ab}{}^c j_{c\,0}$	$[D_a, D_b] = F_{ab}{}^c D_c$
$\sum_R e(R)\lvert\phi_R\rangle$	$\sum_R e(R) Y_R(x^i) := e(x^i)$

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$$E_A{}^I = \begin{pmatrix} e_a{}^i & 0 \\ 0 & e_{\bar{a}}{}^{\bar{i}} \end{pmatrix} \qquad S_{AB} = 2\begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar{a}\bar{b}} \end{pmatrix} \qquad \eta_{AB} = 2\begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta_{\bar{a}\bar{b}} \end{pmatrix}$$

Weak constraint (level matching), later strong constraint

- level matched string field $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ requires

$$(D_a D^a - D_{\bar{a}} D^{\bar{a}}) \cdot = 0 \quad \text{with} \quad \cdot \in \{\epsilon^{a\bar{b}}, e, \bar{e}, f^a, f^{\bar{b}}\}$$

- rewritten in terms of η^{AB} and $D_A = \begin{pmatrix} D_a & D_{\bar{a}} \end{pmatrix}$

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$$\eta^{AB} D_A D_B \cdot = D_A D^A \cdot = 0$$

- change to curved indices using $E_A{}^M$

$$(\partial_M \partial^M - 2\partial_M d \partial^M) \cdot = 0 \qquad \text{with} \qquad d = \phi - \frac{1}{2} \log \sqrt{g}$$

- 

term which is absent in DFT $\to$ adsorb in cov. derivative

$$\boxed{\nabla_M \partial^M \cdot = 0} \quad \text{with} \quad \nabla_M V^N = \partial_M V^N + \Gamma_{MK}{}^N V^K \,, \quad \Gamma_{MK}{}^M = -2\partial_K d$$

Results (leading order k^{-1})

- calculate quadratic and cubic string functions
- integrate out auxiliary fields f^a and $f^{\bar{b}}$
- perform field redefinition

$$(2\kappa^2)S = \int d^{2D}X\sqrt{H}\Big[\frac{1}{4}\epsilon_{a\bar{b}}\Box\epsilon^{a\bar{b}} + \dots$$
$$-\frac{1}{4}\epsilon_{a\bar{b}}\left(F^{ac}{}_{d}\bar{D}^{\bar{e}}\epsilon^{d\bar{b}}\epsilon_{c\bar{e}} + F^{\bar{b}\bar{c}}{}_{\bar{d}}D^{e}\epsilon^{a\bar{d}}\epsilon_{e\bar{c}}\right)$$
$$-\frac{1}{12}F^{ace}F^{\bar{b}\bar{d}\bar{f}}\epsilon_{a\bar{b}}\epsilon_{c\bar{d}}\epsilon_{e\bar{f}} + \dots\Big]$$

- 

terms e.g. potential
- vanish in abelian limit $F_{abc} \to 0$ and $F_{\bar{a}\bar{b}\bar{c}} \to 0$

Gauge transformations

- tree level gauge transformation in CSFT [Zwiebach, 1993]

$$\delta_\Lambda |\Psi\rangle = Q|\Lambda\rangle + [\Lambda, \Psi]_0 + \cdots$$

- string field for gauge parameter [Hull and Zwiebach, 2009]

$$|\Lambda\rangle = \sum_R \Big[\frac{1}{2}\lambda^a(R) j_{a\,-1} c_1 - \frac{1}{2}\lambda^{\bar{b}}(R) j_{\bar{b}\,-1} \bar{c}_1 + \mu(R) c_0^+\Big] |\phi_R\rangle$$

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- after field redefinition and μ gauge fixing

$$\begin{aligned}\delta_\lambda \epsilon_{a\bar{b}} =& D_{\bar{b}}\lambda_a + \frac{1}{2}\big[D_a \lambda^c \epsilon_{c\bar{b}} - D^c \lambda_a \epsilon_{c\bar{b}} + \lambda_c D^c \epsilon_{a\bar{b}} + F_{ac}{}^d \lambda^c \epsilon_{d\bar{b}}\big] \\ & D_a \lambda_{\bar{b}} + \frac{1}{2}\big[D_{\bar{b}} \lambda^{\bar{c}} \epsilon_{a\bar{c}} - D^{\bar{c}} \lambda_{\bar{b}} \epsilon_{a\bar{c}} + \lambda_{\bar{c}} D^{\bar{c}} \epsilon_{a\bar{b}} + F_{\bar{b}\bar{c}}{}^{\bar{d}} \lambda^{\bar{c}} \epsilon_{a\bar{d}}\big]\end{aligned}$$

$$\delta_\lambda d = -\frac{1}{4} D_a \lambda^a + \frac{1}{2} \lambda_a D^a d - \frac{1}{4} D_{\bar{a}} \lambda^{\bar{a}} + \frac{1}{2} \lambda_{\bar{a}} D^{\bar{a}} d$$

Doubled objects

promising results, but bulky

Rewrite action/gauge trafo in terms of doubled object

+ simplifies expressions considerably
+ extrapolation from cubic to all order in fields

Doubled objects

object	doubled version
η_{ab}, $\eta_{\bar{a}\bar{b}}$	$\eta_{AB} = 2\begin{pmatrix}\eta_{ab} & 0 \\ 0 & -\eta_{\bar{a}\bar{b}}\end{pmatrix}$ $\quad S_{AB} = 2\begin{pmatrix}\eta_{ab} & 0 \\ 0 & \eta_{\bar{a}\bar{b}}\end{pmatrix}$
$e_a{}^i$, $e_{\bar{a}}{}^{\bar{i}}$	$E_A{}^I = \begin{pmatrix}e_a{}^i & 0 \\ 0 & e_{\bar{a}}{}^{\bar{i}}\end{pmatrix}$
D_a, $D_{\bar{a}}$	$D_A = \begin{pmatrix}D_a & D_{\bar{a}}\end{pmatrix} = E_A{}^I \partial_I$ with $\partial_I = \begin{pmatrix}\partial_i & \partial_{\bar{i}}\end{pmatrix}$

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${e_a}^i$, ${e_{\bar{a}}}^{\bar{i}}$	${E_A}^I = \begin{pmatrix}{e_a}^i & 0 \\ 0 & {e_{\bar{a}}}^{\bar{i}}\end{pmatrix}$
D_a, $D_{\bar{a}}$	$D_A = \begin{pmatrix}D_a & D_{\bar{a}}\end{pmatrix} = {E_A}^I \partial_I$ with $\partial_I = \begin{pmatrix}\partial_i & \partial_{\bar{i}}\end{pmatrix}$
ξ^i, $\xi^{\bar{i}}$	$\xi^I = \begin{pmatrix}\xi^i & \xi^{\bar{i}}\end{pmatrix}$
${F_{ab}}^c$, ${F_{\bar{a}\bar{b}}}^{\bar{c}}$	${F_{AB}}^C = \begin{cases}{F_{ab}}^c \\ {F_{\bar{a}\bar{b}}}^{\bar{c}} \\ 0 & \text{otherw.}\end{cases}$ $\qquad [D_A, D_B] = {F_{AB}}^C D_C$

Gauge transformations

- “doubled” version of fluctuations $\epsilon^{a\bar{b}}$

$$\epsilon^{AB} = \begin{pmatrix} 0 & -\epsilon^{a\bar{b}} \\ -\epsilon^{\bar{a}b} & 0 \end{pmatrix} \quad \text{with} \quad \epsilon^{a\bar{b}} = (\epsilon^T)^{\bar{b}a}$$

- generate generalized metric [Hohm, Hull, and Zwiebach, 2010]

$$\mathcal{H}^{AB} = S^{AB} + \epsilon^{AB} + \frac{1}{2}\epsilon^{AC} S_{CD} \epsilon^{DB} + \dots = \exp(\epsilon^{AB})$$

with the defining property $\mathcal{H}^{AC}\eta_{CD}\mathcal{H}^{DB} = \eta^{AB}$

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- generalized Lie derivative [Hull and Zwiebach, 2009,Grana and Marques, 2012]

$$\mathcal{L}_\lambda \mathcal{H}^{AB} = \lambda^C D_C \mathcal{H}^{AB} + (D^A \lambda_C - D_C \lambda^A)\mathcal{H}^{CB} + (D^B \lambda_C - D_C \lambda^B)\mathcal{H}^{AC} + F^A{}_{CD}\lambda^C \mathcal{H}^{DB} + F^B{}_{CD}\lambda^C \mathcal{H}^{AD}$$

- setting $\delta_\lambda S^{AB} := 0$ and using

$$\delta_\lambda \epsilon^{AB} = \mathcal{L}_\lambda S^{AB} + \mathcal{L}_\lambda \epsilon^{AB} + \mathcal{L}_\lambda S^{(A}{}_C S^{B)}{}_D \epsilon^{CD} .$$

results in $\boxed{\delta_\lambda \mathcal{H}^{AB} = \mathcal{L}_\lambda \mathcal{H}^{AB} + \mathcal{O}(\epsilon^2)}$

- similar for the generalized dilaton d

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results in $$\boxed{\delta_\lambda \mathcal{H}^{AB} = \mathcal{L}_\lambda \mathcal{H}^{AB} + \mathcal{O}(\epsilon^2)}$$

- similar for the generalized dilaton d
- introduce covariant derivative

$$\nabla_A V^B = D_A V^B + \frac{1}{3} F^B{}_{AC} V^C$$

- 

generalized Lie derivative, e.g. for vector

$$\mathcal{L}_\lambda V^A = \lambda^B \nabla_B V^A + (\nabla^A \lambda_B - \nabla_B \lambda^A) V^B \quad \text{instead of}$$

$$\mathcal{L}_\lambda V^I = \lambda^J \partial_J V^I + (\partial^I \lambda_J - \partial_J \lambda^I) V^J \quad \text{in traditional DFT}$$

Gauge algebra

- CSFT to cubic order fulfills

$$\delta_{\Lambda_1}\delta_{\Lambda_2} - \delta_{\Lambda_2}\delta_{\Lambda_1} = \delta_{\Lambda_{12}} \quad \text{with} \quad \Lambda_{12} = [\Lambda_2, \Lambda_1]_0$$

- after field redefinition and μ fixing $\lambda_{12}^A = [\lambda_2, \lambda_1]_{\mathrm{C}}^A$ with

$$[\lambda_1, \lambda_2]_C^A = \lambda_1^B \nabla_B \lambda_2^A - \frac{1}{2}\lambda_1^B \nabla^A \lambda_{2\,B} - (1 \leftrightarrow 2)$$

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- algebra closes up to a trivial gauge transformation if
  1. fluctuations and parameter fulfill NEW! strong constraint $D_A D^A \cdot$
  2. background fulfills Jacobi identity

$$F_{E[AB}F^E{}_{C]D} = 0$$

- no strong constraint required for background

Action

$$X^I = \begin{pmatrix} x^i & x^{\bar{i}} \end{pmatrix}$$

$$S = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

Action

$$X^I = \begin{pmatrix} x^i & x^{\bar{i}} \end{pmatrix}$$

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$$\mathcal{R} = 4\mathcal{H}^{MN}\nabla_M\nabla_N d - \nabla_M\nabla_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\nabla_M d\nabla_N d + 4\nabla_M\mathcal{H}^{MN}\nabla_N d$$
$$+ \frac{1}{8}\mathcal{H}^{MN}\nabla_M\mathcal{H}^{KL}\nabla_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\nabla_N\mathcal{H}^{KL}\nabla_L\mathcal{H}_{MK} + \frac{1}{6}F_{MKL}F_N{}^{KL}H^{MN}$$

Action

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$$\mathcal{R} = 4\mathcal{H}^{MN}\nabla_M\nabla_N d - \nabla_M\nabla_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\nabla_M d\nabla_N d + 4\nabla_M\mathcal{H}^{MN}\nabla_N d + \frac{1}{8}\mathcal{H}^{MN}\nabla_M\mathcal{H}^{KL}\nabla_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\nabla_N\mathcal{H}^{KL}\nabla_L\mathcal{H}_{MK} + \frac{1}{6}F_{MKL}F_N{}^{KL}H^{MN}$$

- lower indices with $\eta_{MN} = E^A{}_M E^B{}_N \eta_{AB} \neq$ const.
- $H_{IJ} = E^A{}_M E^B{}_N S_{AB}$ background generalized metric

$$\nabla_M d = \partial_M \tilde{d}$$

$$\nabla_M \mathcal{H}^{KL} = \partial_M \mathcal{H}^{KL} + \Gamma_{MJ}{}^K \mathcal{H}^{JL} + \Gamma_{MJ}{}^L \mathcal{H}^{KJ}$$

Symmetries

- generalized diffeomorphisms (only under strong constraint)

$$\delta_\xi \mathcal{H}^{MN} = \mathcal{L}_\xi \mathcal{H}^{MN}$$

$$\delta_\xi d = \mathcal{L}_\xi d = \xi^M \nabla_M d + \frac{1}{2} \nabla_M \xi^M$$

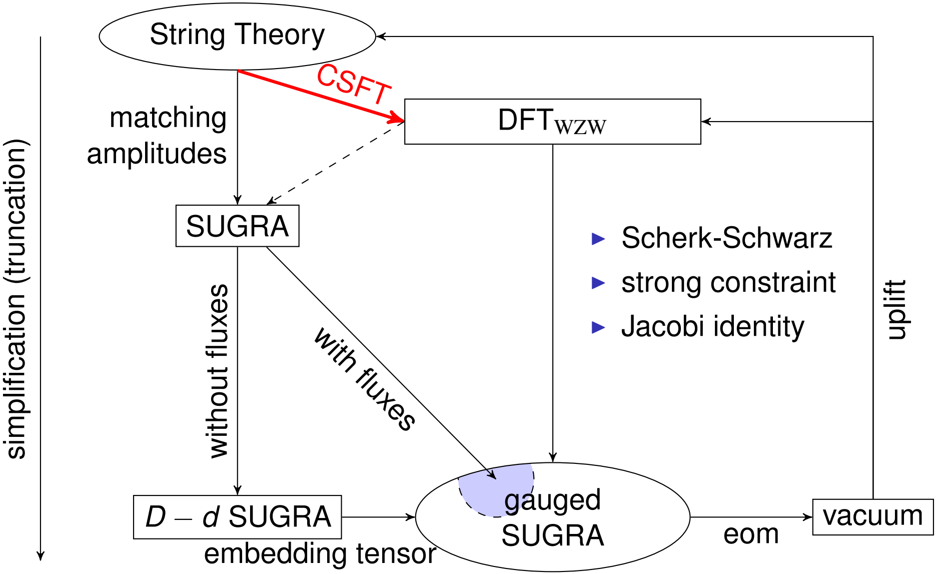
NEW! $2D$-diffeomorphisms (always)

$$\delta_\xi \mathcal{H}^{MN} = L_\xi \mathcal{H}^{MN} = \xi^I \partial_I \mathcal{H}^{MN} + \mathcal{H}^{IN} \partial_I \xi^M + \mathcal{H}^{MI} \partial_I \xi^N$$

$$\delta_\xi e^{-2d} = L_\xi e^{-2d} = \partial_I (\xi^I e^{-2d})$$

- $\left.\begin{matrix} e^{-2d} \\ \mathcal{R} \end{matrix}\right\}$ transform as $\left\{\begin{matrix} +1 \text{ desity} \\ \text{scalar} \end{matrix}\right\}$ under gen. and $2D$-diff.

Reminder: Generalized Scherk-Schwarz compactification



- Scherk-Schwarz
- strong constraint
- Jacobi identity

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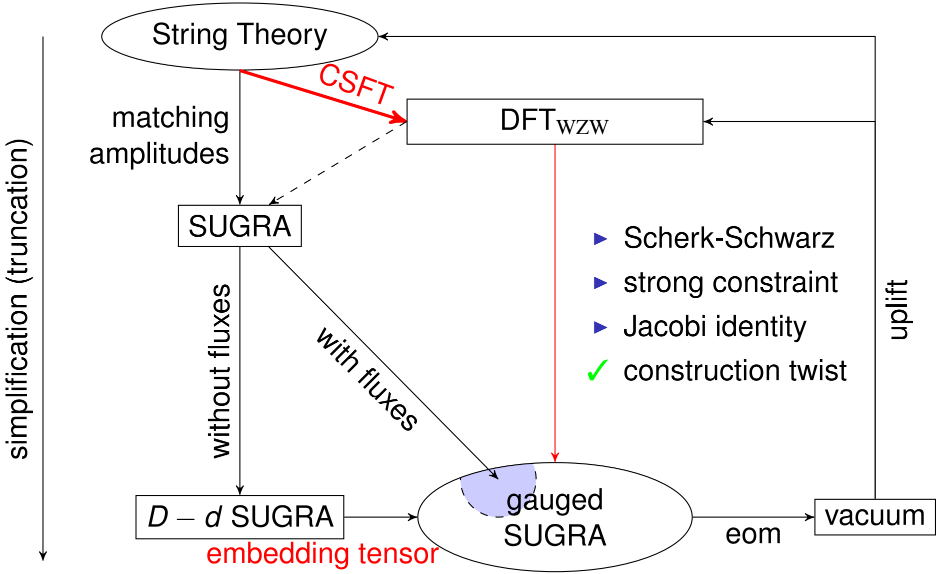
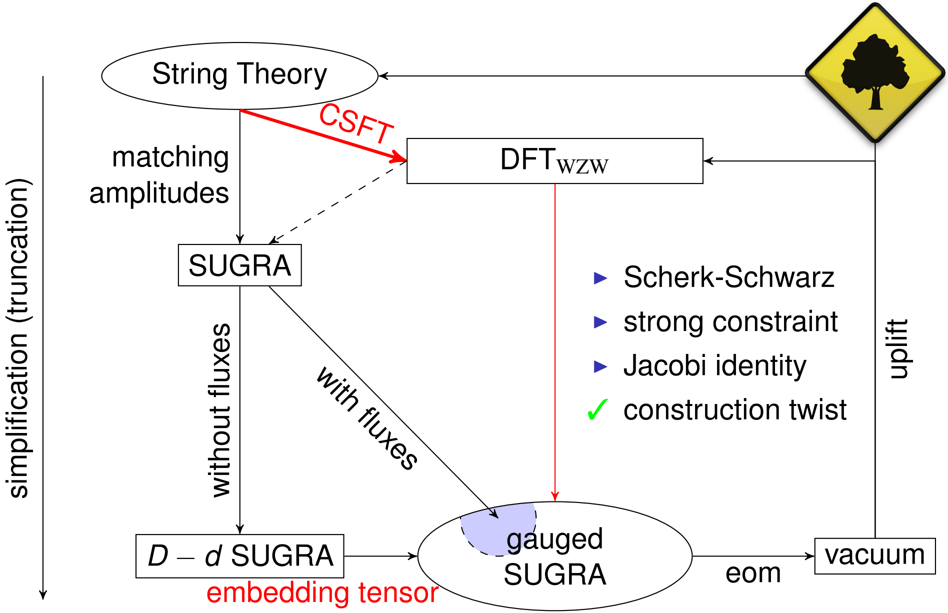


- Scherk-Schwarz
- strong constraint
- Jacobi identity
- ✓ construction twist

Reminder: Generalized Scherk-Schwarz compactification



Embedding tensor

ID	$M_{mn}/\ \cos\alpha$	$\tilde{M}^{mn}/\ \sin\alpha$	range of α	gauging
1	$\mathrm{diag}(1,1,1,1)$	$\mathrm{diag}(1,1,1,1)$	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4)\,, & \alpha \neq \frac{\pi}{4}\,, \\ \mathrm{SO}(3)\,, & \alpha = \frac{\pi}{4}\,. \end{cases}$
2	$\mathrm{diag}(1,1,1,-1)$	$\mathrm{diag}(1,1,1,-1)$	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\mathrm{SO}(3,1)$
[illegible]	(1, 1 [illegible]	diag([illegible] 1)	[illegible] < $\frac{\pi}{4}$	[illegible]O(2,2) , α [illegible] $\frac{\pi}{4}$, [illegible]

[Dibitetto, Fernandez-Melgarejo, Marques, and Roest, 2012]

- fluxes for embedding one

$$F_{abc} = \sqrt{2}\epsilon_{abc}(\cos\alpha + \sin\alpha) \quad \text{and} \quad F_{\bar{a}\bar{b}\bar{c}} = \sqrt{2}\epsilon_{abc}(\cos\alpha - \sin\alpha)$$

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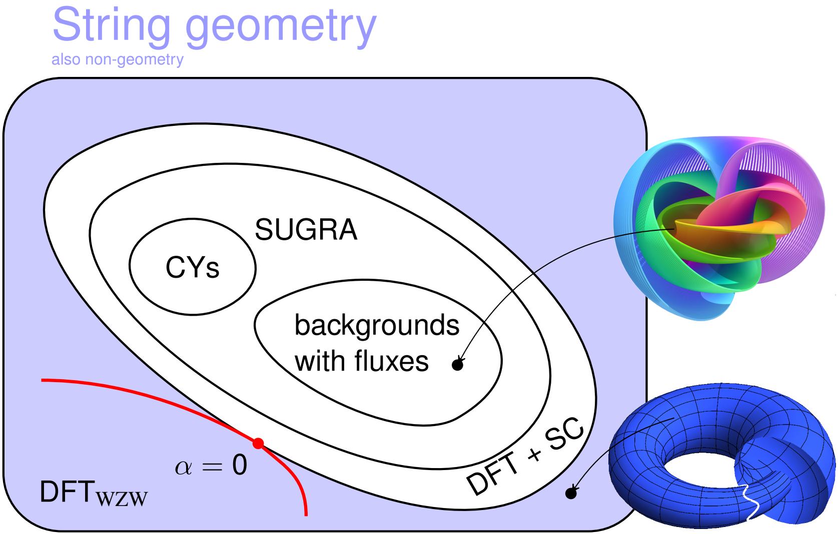
- DFT strong constraint holds only for

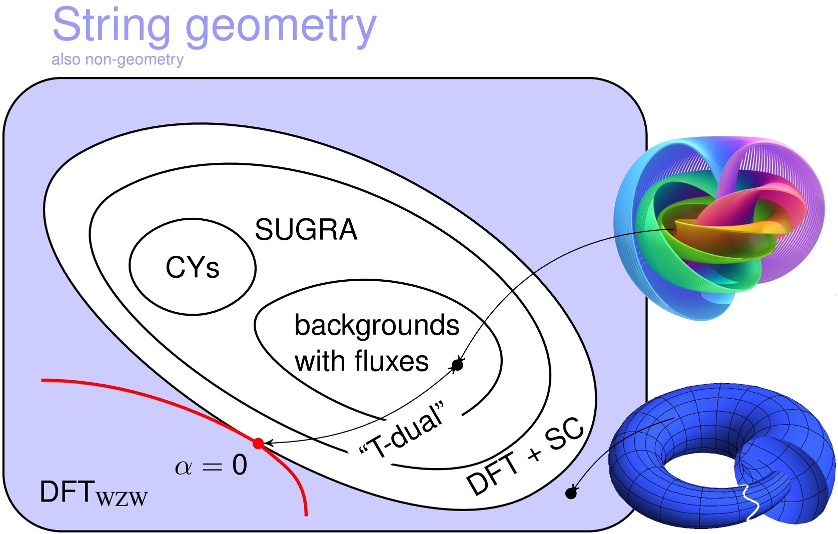
$$F_{ABC}F^{ABC} = 6\sin(2\alpha) = 0 \quad \to \alpha = \frac{\pi}{2}n \quad n \in \mathbb{Z}$$

- Jacobi identity holds always

The landscape again



The landscape again



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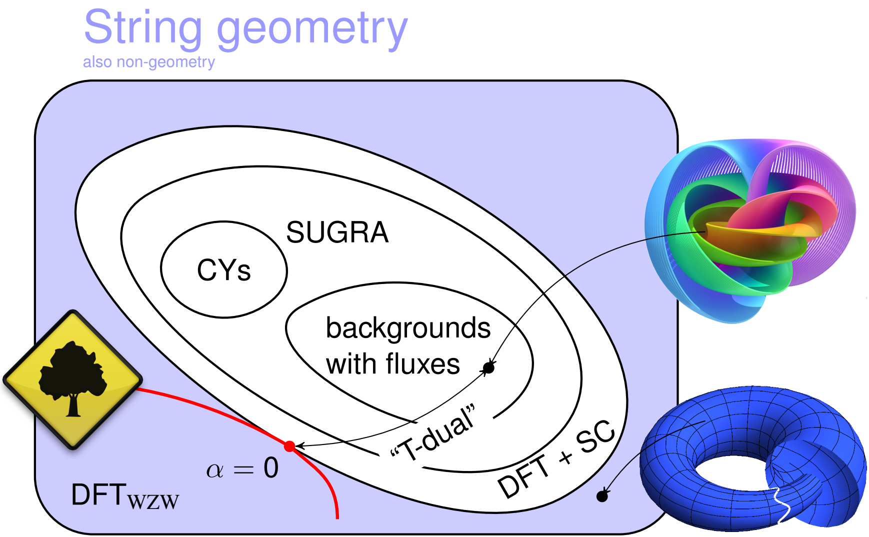

Summary

DFT_{WZW} is a generalization of DFT

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- basic idea: go beyond the torus

strong constraint, symmetries, action

genuinely non-geometric backgrounds and twist

- DFT arises under the optional extended strong constraint

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- basic idea: go beyond the torus

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- DFT arises under the optional extended strong constraint

Todo

- find solutions of the new strong constraint

$\rightarrow$ key to understand T-duality in DFT_{WZW}

- α' corrections (here k^{-2}, k^{-3}, $\dots$) [Hohm, Siegel, and Zwiebach, 2013]
- coset and orbifold CFTs

Thank you for your attention. Are there any questions?

Covariant derivative [Cederwall, 2014]

- non-vanishing torsion and Riemann curvature

$$[\nabla_A, \nabla_B] V_C = R_{ABC}{}^D V_D - T^D{}_{AB} \nabla_D V_C \quad \text{with}$$

$$T^A{}_{BC} = -\frac{1}{3} F^A{}_{BC} \quad \text{and} \quad R_{ABC}{}^D = \frac{2}{9} F_{AB}{}^E F_{EC}{}^D$$

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- compatible with $E_A{}^I$, η_{AB} and S_{AB}

$$\nabla_C E_A{}^I = \nabla_C \eta_{AB} = \nabla_C S_{AB} = 0$$

- compatible with partial integration

$$\int d^{2D} X e^{-2d} U \nabla_M V^M = -\int d^{2D} X e^{-2d} \nabla_M U V^M$$

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- compatible with partial integration

$$\int d^{2D} X e^{-2d} U \nabla_M V^M = - \int d^{2D} X e^{-2d} \nabla_M U V^M$$

- non-vanishing generalized torsion