

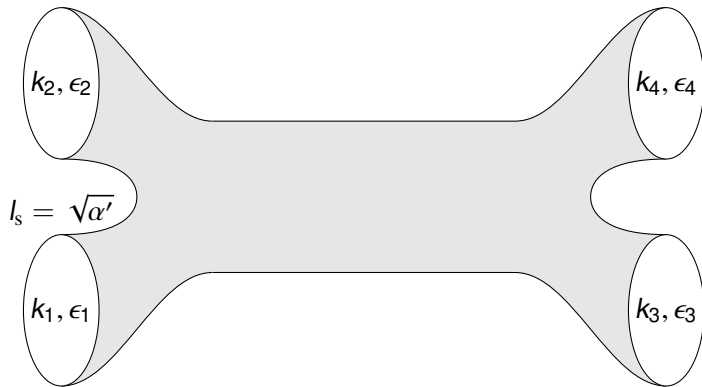
The generalized Cartan Geometry of α' -corrections Falk Hassler

Based on work with Martin Cederwall, Achilleas Gitsis,
Ondřej Hulík, David Osten, Yuho Sakatani and Luca Scala

String amplitudes, Effective Field Theory and gravity

massless modes

metric	g_{ij}
gauge potential	B_{ij}
dilaton	ϕ
$\vdots$	

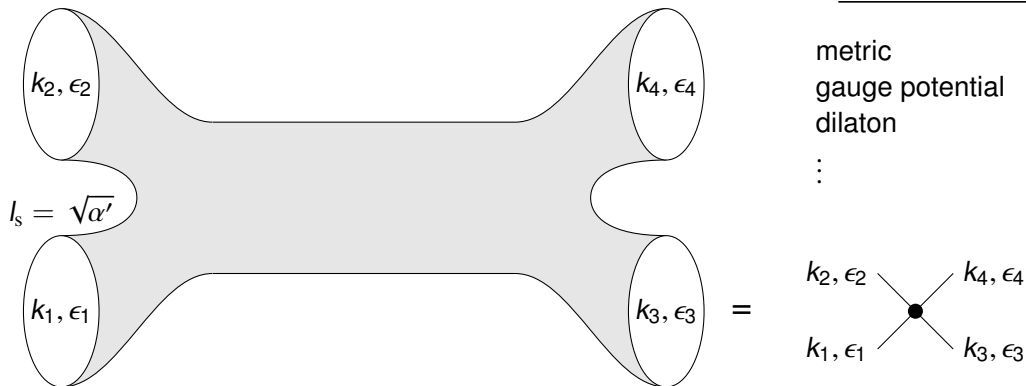


$$\begin{aligned}\mathcal{A}^{\text{tree}}(s, t, u) &= g_s^2 \frac{(\alpha')^4}{stu} \frac{\Gamma(1 - \alpha' s) \Gamma(1 - \alpha' t) \Gamma(1 - \alpha' u)}{\Gamma(1 + \alpha' s) \Gamma(1 + \alpha' t) \Gamma(1 + \alpha' u)} \mathcal{R}^4 \\ &= g_s^2 (\alpha')^2 \left[\frac{1}{stu} + 2\zeta(3)(\alpha')^3 + \zeta(5)(\alpha')^5 (s^2 + t^2 + u^2) + \dots \right] \mathcal{R}^4\end{aligned}$$

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String amplitudes, Effective Field Theory and gravity

1. select relevant degrees of freedom

$$(g_{ij}, B_{ij}, \phi)$$

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- ▶ diffeomorphisms, and
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3. expand action by writing all terms allowed by symmetries (α')

$$R_{ijk}{}^l, \quad H_{ijk} = 3\partial_{[i}B_{jk]}, \quad \phi, \quad \nabla_i \quad \text{building blocks}$$

$$S = \frac{2\pi}{(4\pi^2\alpha')^4} \int dx^d \sqrt{-g} e^{-2\phi} \left[c_1 R + c_2 \nabla_i \phi \nabla^i \phi + c_3 H_{ijk} H^{ijk} + \alpha'(\dots) + \dots \right]$$

String amplitudes, Effective Field Theory and gravity

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 $R_{ijk}{}^l, \quad H_{ijk} = 3\partial_{[i}B_{jk]}, \quad \phi, \quad \nabla_i$ building blocks
4. fix their coefficients by matching the amplitudes

$$S = \frac{2\pi}{(4\pi^2\alpha')^4} \int dx^d \sqrt{-g} e^{-2\phi} \left[R + 4\nabla_i \phi \nabla^i \phi + \frac{1}{12} H_{ijk} H^{ijk} + \alpha'(\dots) + \dots \right]$$

Challenges

► explosion of terms

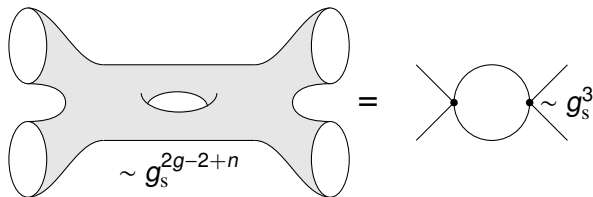
	coeff.	theories
α'^0	3	all
α'^1	8	bos., het.
α'^2	60	bos., het.
α'^3	872	all

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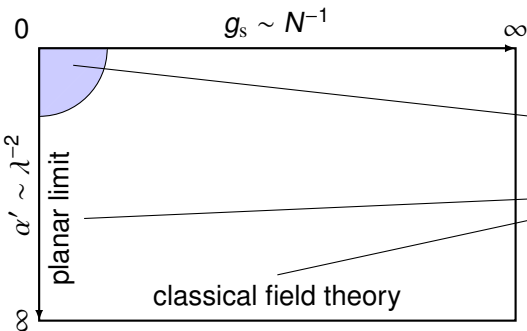
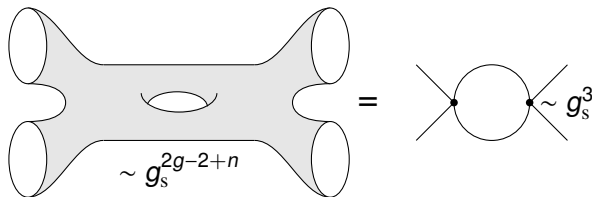


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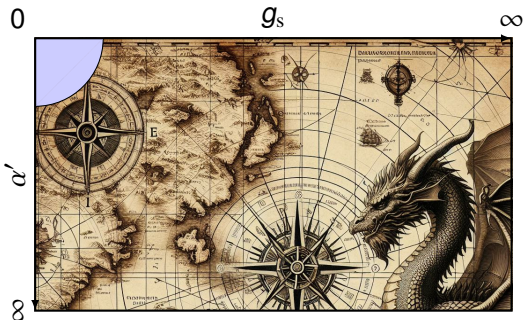
Expansion in α' and g_s ! We know only

- ▶ leading orders in supergravity
- ▶ thanks to holography a bit more

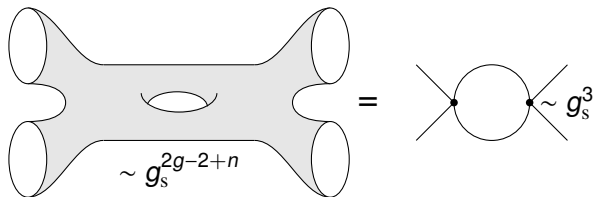
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Expansion in α' and g_s ! We know only

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How to go beyond?

Idea: new symmetries

1. select relevant degrees of freedom
2. identify their symmetries^{*)}
3. ...

additional symmetries

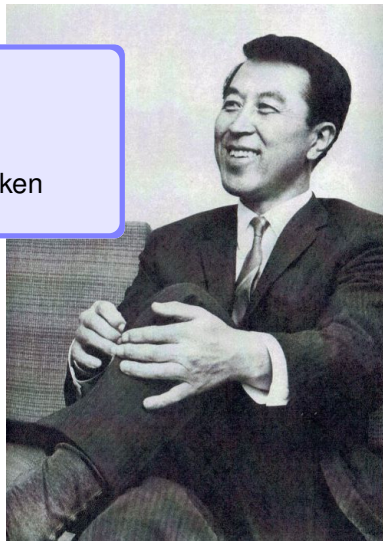
- ▶ fermionic
- ▶ hidden
- ▶ spontaneously broken

Idea: spontaneously broken symmetries

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Yoichiro Nambu, 1965

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- ▶ hidden
- ▶ spontaneously broken

Coset construction (non-linear realization)

- ▶ G = the full symmetry group
- ▶ H = residual symmetries after spontaneous breaking
- ▶ introduce Maurer-Cartan form $\Omega = g^{-1}dg$ for $g \in G/H$
- ▶ expand it in broken generators t_a as $\Omega = \Omega^a t_a + \dots$

Lagrangian

$$L = L(\Omega^a, D_a)$$

Hypothesis

α' -corrections in supergravity might be governed by the infinite dimensional $G=\text{Poincaré}_\infty$ group.

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- ▶ What is this group?

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- ▶ HOW?

By having some kind of a coset construction!

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Batalin-Vilkovisky (BV) formalism

- ▶ HOW?

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Cartan geometry

Degrees of freedom in linearized gravity. Part I: the complex

► frame field

$$e_i^a = \delta_i^a + \varepsilon_i^a$$

► symmetries

$$\delta \varepsilon^a = d\xi^a + \Lambda^a_b dx^b$$

useful relations

$$g_{ij} = \eta_{ab} e_i^a e_j^b$$

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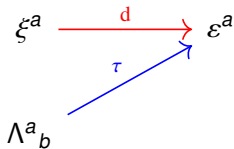
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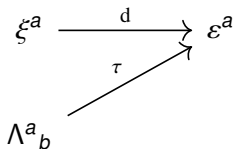
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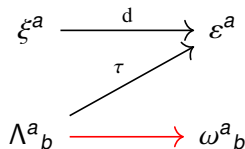
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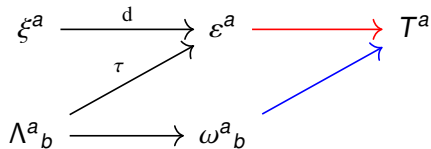
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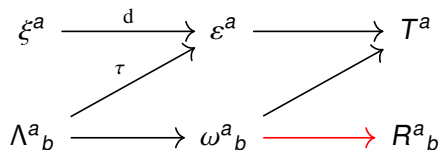
$$\delta \omega^a_b = d\Lambda^a_b$$

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$$R^a_b = d\omega^a_b$$



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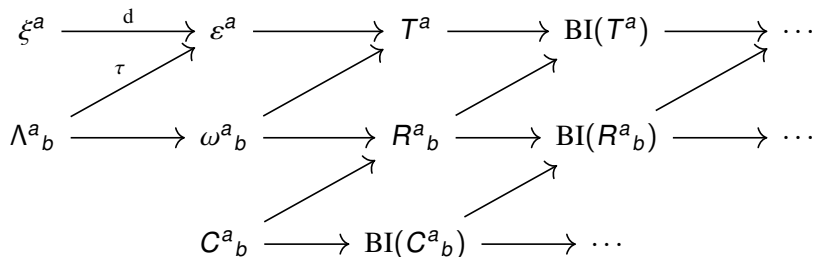
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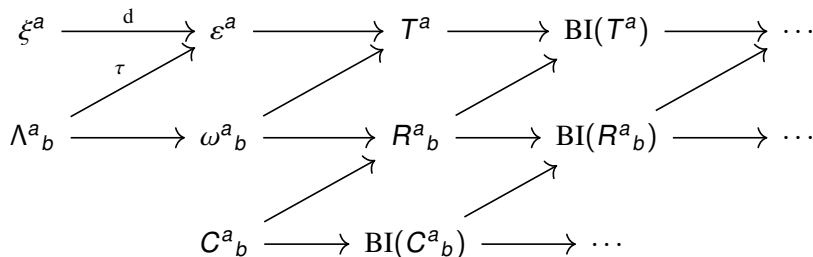
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$$d^2 = 0$$

$$\tau^2 = 0$$

$$d \circ \tau + \tau \circ d = 0$$

Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

$$\begin{array}{ccccccc}
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 \end{array}$$

Note: A blue arrow points from Λ^a_b to ε^a in the original image.

- compute cohomology for the exact sequence

$$0 \longrightarrow \Lambda_{ab} \xrightarrow{\tau} \varepsilon_{ai} \longrightarrow 0$$

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Λ_{ab} is represented by a vertical box with two compartments.
 ε_{ai} is represented by a vertical box with two compartments plus a horizontal box with two compartments.

Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

$$\begin{array}{ccccccc}
 \xi^a & \xrightarrow{d} & \boxed{\begin{array}{|c|c|} \hline & \bullet \\ \hline \end{array}} & \longrightarrow & T^a & \longrightarrow & \text{BI}(T^a) \longrightarrow \dots \\
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Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

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 \end{array}$$

A blue arrow points from ω^a_b to T^a .

- compute cohomology for all diagonal exact sequences

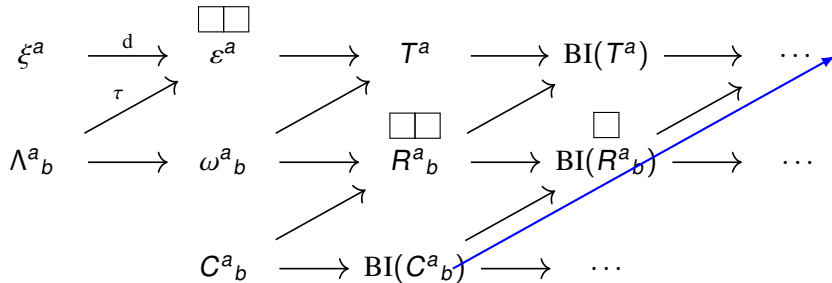
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 & & & & & & \\
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A blue diagonal arrow points from C^a_b to $\text{BI}(T^a)$.

- compute cohomology for all diagonal exact sequences
- only with Weyl tensor cohomology classes of ε^a and R^a_b match

Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]



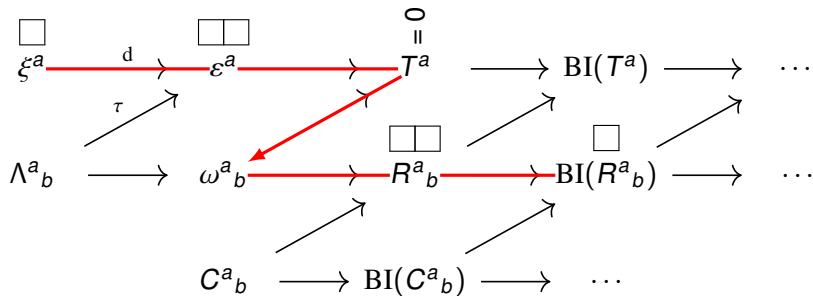
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Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

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 & \nearrow \tau & & \nearrow & & \nearrow & \\
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Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]



- ▶ compute cohomology for all diagonal exact sequences
- ▶ only with Weyl tensor cohomology classes of ε^a and R^a_b match
- ▶ after imposing torsion constraint ($T^a = 0$), we get

gauge $\longrightarrow$ dof $\longrightarrow$ eom $\longrightarrow$ Bianchi

The background independent version and Cartan geometry

- ▶ idea: compress complex into a chain

$$\begin{array}{ccccccccc} \xi^a & \xrightarrow{d} & \varepsilon^a & \longrightarrow & T^a & \longrightarrow & \text{BI}(T^a) & \longrightarrow & \dots \\ & \nearrow \tau & & & \nearrow & & \nearrow & & \\ \Lambda^a_b & \longrightarrow & \omega^a_b & \longrightarrow & R^a_b & \longrightarrow & \text{BI}(R^a_b) & \longrightarrow & \dots \\ & & \nearrow & & \nearrow & & \nearrow & & \\ & & C^a_b & \longrightarrow & \text{BI}(C^a_b) & \longrightarrow & \dots & & \end{array}$$

The background independent version and Cartan geometry

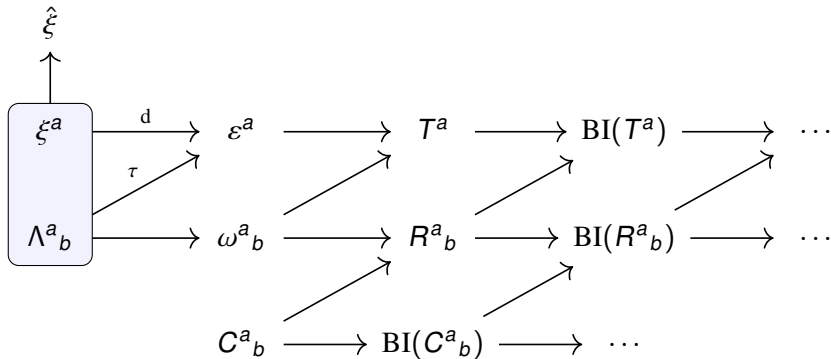
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$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}$$

Poincaré algebra

P_a translations

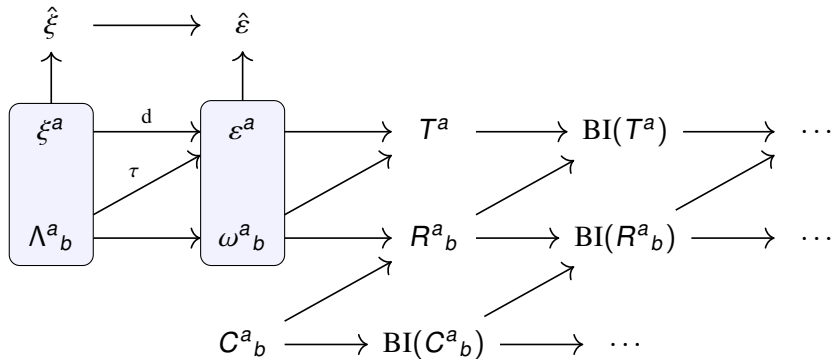
M_{ab} Lorentz transformations



The background independent version and Cartan geometry

- idea: compress complex into a chain

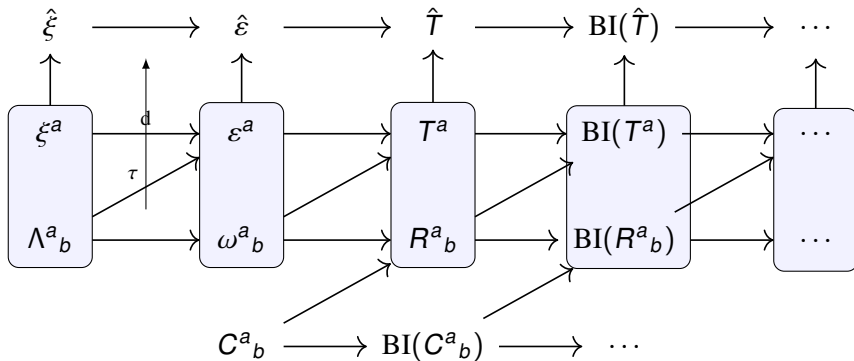
$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}, \quad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}$$



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- ▶ and use exterior derivative

$$\begin{aligned} \hat{d} &= d + \hat{e} \wedge && \text{with} \\ \hat{e} &= e_i^a P_a dx^i + \omega^{ab} M_{ab} && \underline{\text{Cartan connection}} \end{aligned}$$

$$\hat{\xi} \xrightarrow{\hat{d}} \hat{\varepsilon} \longrightarrow \hat{T} \longrightarrow \text{BI}(\hat{T}) \longrightarrow \dots$$

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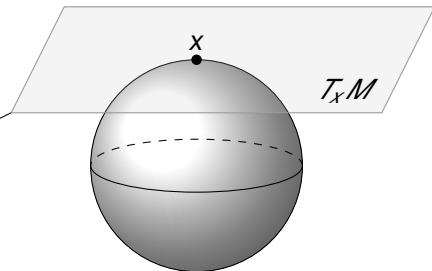
with

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Cartan connection

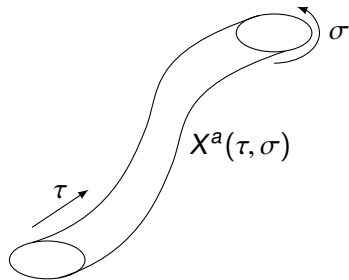
- ▶ substituting $\hat{\varepsilon} \rightarrow \hat{e}$ in the chain

- ▶ only input is model space G/H



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Where is the Poincaré_∞?



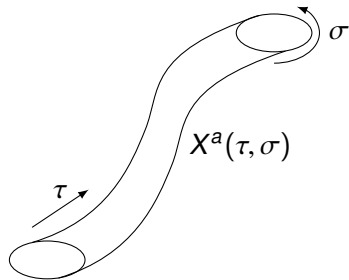
minimal coupling

$$- \int B, \quad B = \frac{1}{2} B_{ab} dx^a \wedge dx^b$$

three-form field strength

$$H = dB$$

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- gauge transformation

$$\delta B = d\varphi + L_\xi B$$

diffeomorphism

- gauge transformation for the gauge transformation

$$\delta\varphi = d\chi$$

Where is the Poincaré_∞?

$$\begin{array}{ccc} \bullet & & \square + \overline{\square} \\ \chi & \xrightarrow{D} & \xi_A \end{array}$$

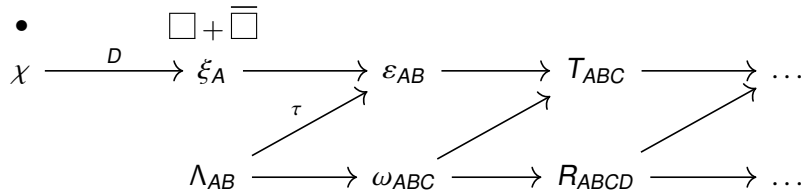
- gauge transformation

$$\delta B = d\varphi + L_\xi B, \quad \text{with combined parameter} \quad \xi^A = \begin{pmatrix} \xi^a & \varphi_a \end{pmatrix}$$

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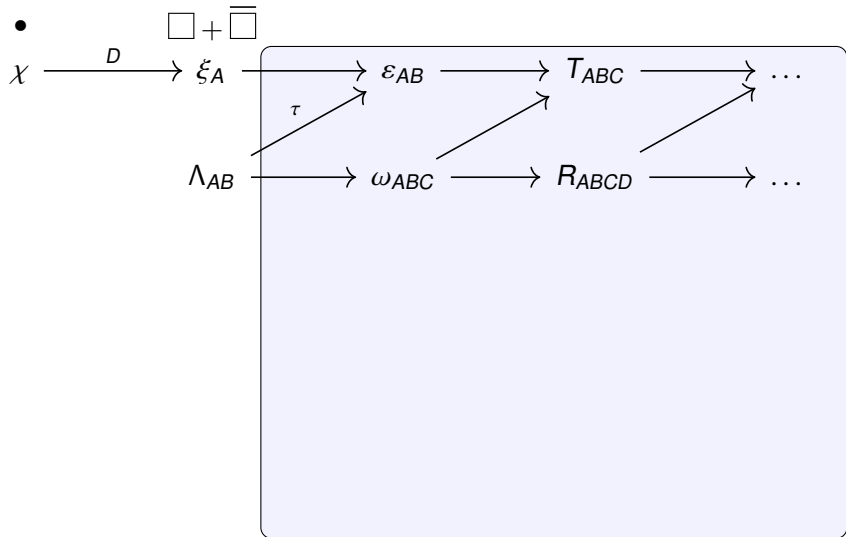
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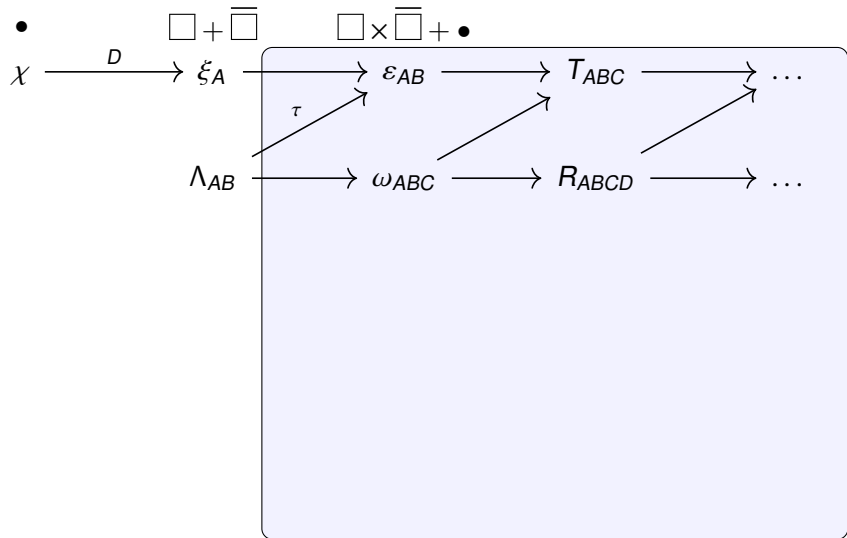
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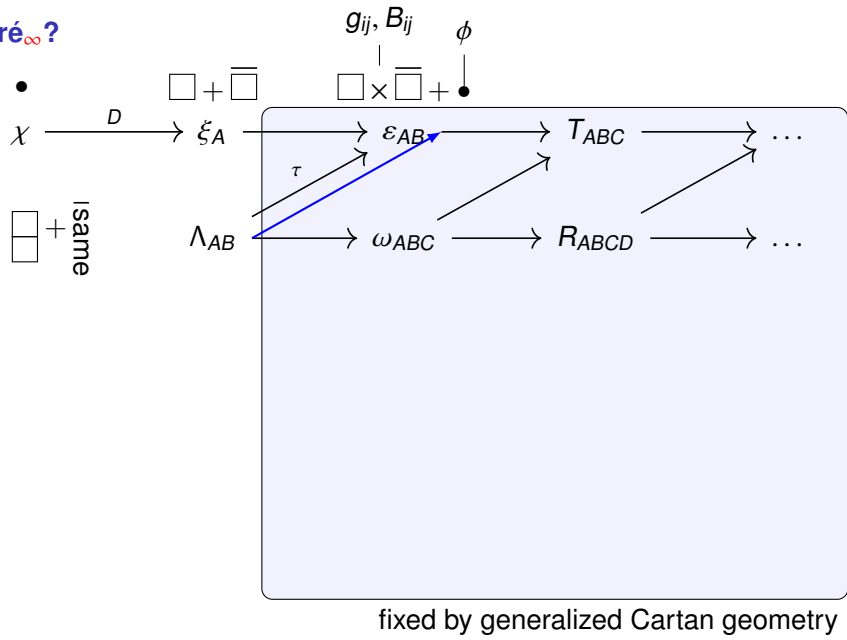
fixed by generalized Cartan geometry

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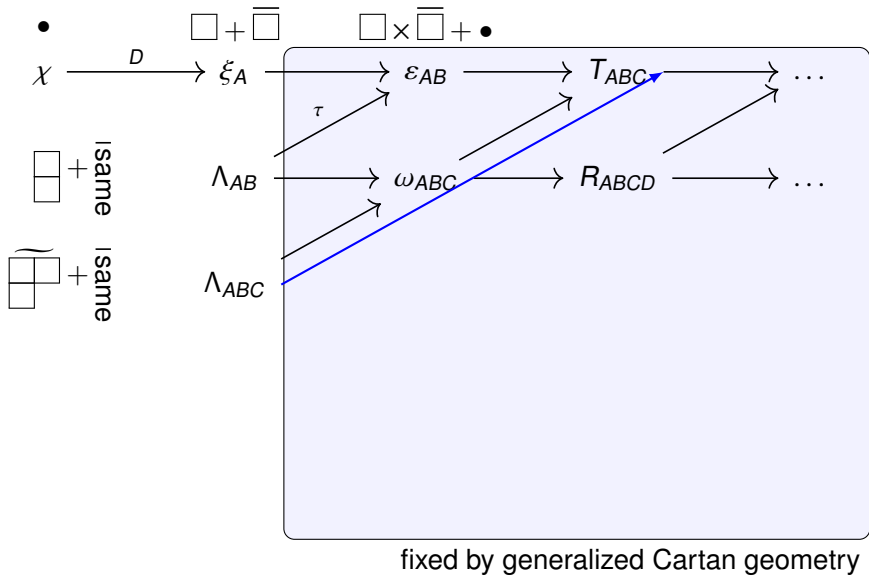


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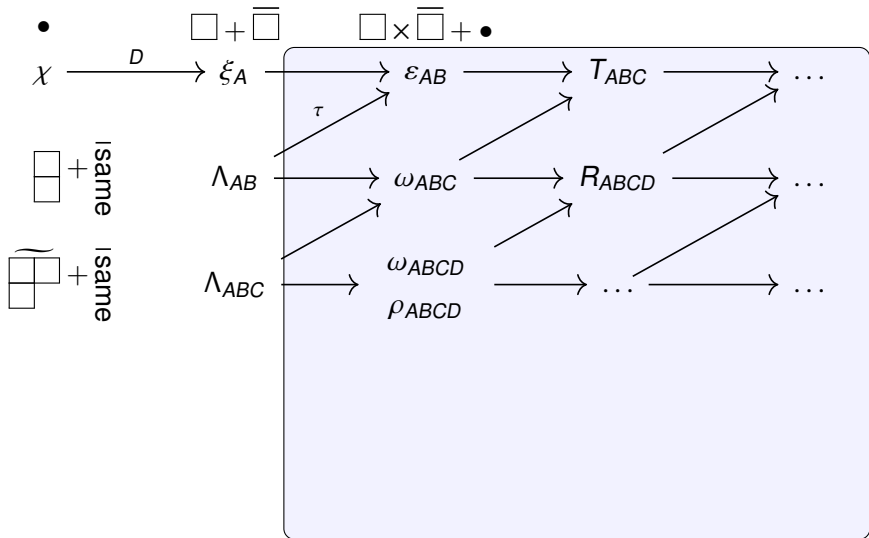
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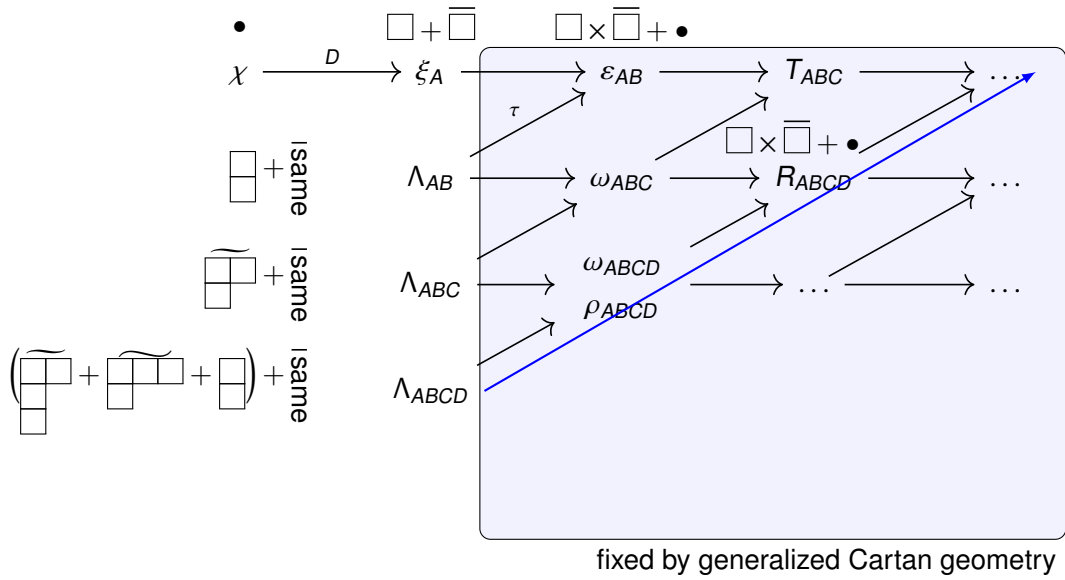


Where is the Poincaré_∞?

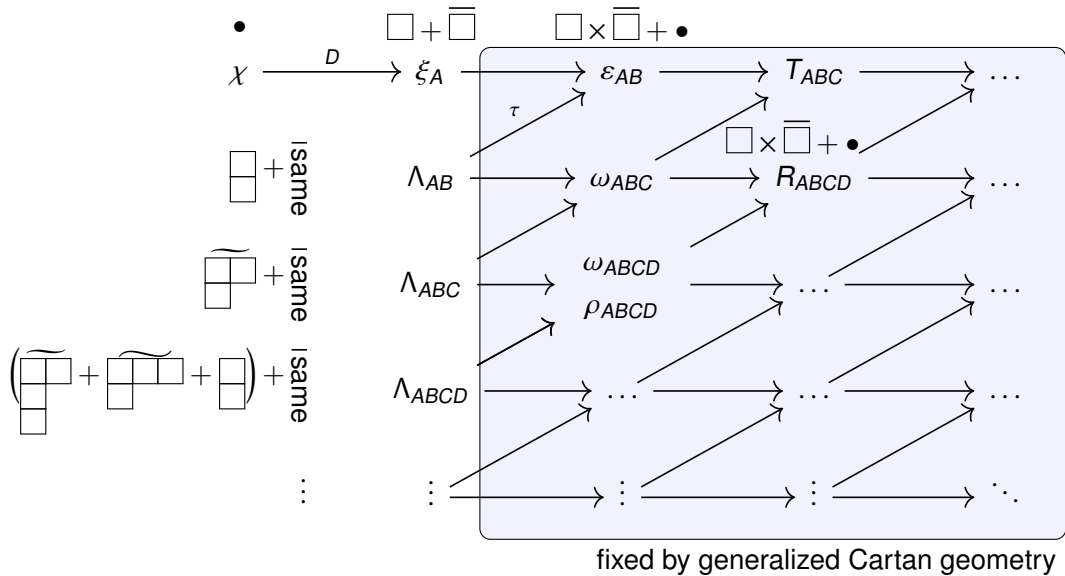


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Where is the Poincaré_∞?



Relevant features of generalized Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulík, Osten 24]

$$\begin{array}{ccccccc} \hat{\mathcal{X}} & \longrightarrow & \hat{\xi} & \xrightarrow{\hat{D}} & \hat{E} & \longrightarrow & \hat{T} \longrightarrow \text{BI}(\hat{T}) \longrightarrow \dots \\ & & \swarrow \quad \searrow & & \swarrow \quad \downarrow \quad \searrow & & \\ & & \xi \quad \Lambda & & E \quad \omega \quad \rho & & \end{array}$$

- new connection ρ with corresponding curvature

Relevant features of generalized Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulik, Osten 24]

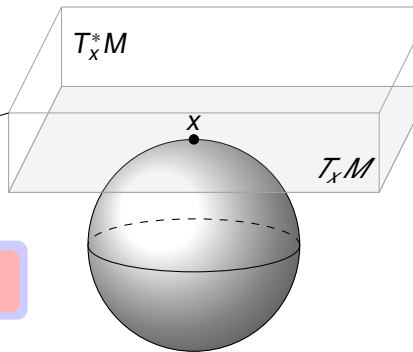
$$\hat{\chi} \longrightarrow \hat{\xi} \xrightarrow{\hat{D}} \hat{E} \longrightarrow \hat{T} \longrightarrow \text{BI}(\hat{T}) \longrightarrow \dots$$

$\hat{\xi} \swarrow \searrow$
 $\xi \quad \Lambda$

$\hat{E} \swarrow \searrow$
 $E \quad \omega \quad \rho$

► new connection ρ with corresponding curvature

► model space is double coset $\tilde{H} \backslash G / H$ generated by



$\dots \quad \tilde{t}^{ABC} \quad \tilde{t}^{AB}$

P_A

$t_{AB} \quad t_{ABC} \quad \dots$

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ξ
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E
 ω

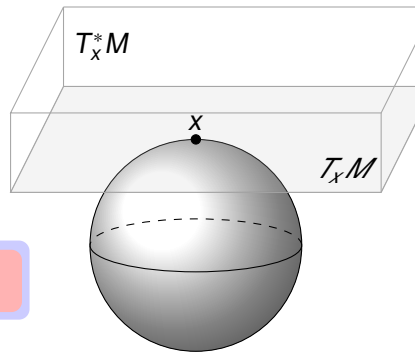
ρ

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$$\underbrace{\dots \quad \tilde{t}^{ABC} \quad \tilde{t}^{AB}}_{\text{fixed by } \tau^2 = 0 \text{ \& cohomology}} \quad P_A \quad \underbrace{t_{AB} \quad t_{ABC} \quad \dots}_{\text{fixed by } \tau^2 = 0 \text{ \& cohomology}}$$

fixed by $\tau^2 = 0$ & cohomology



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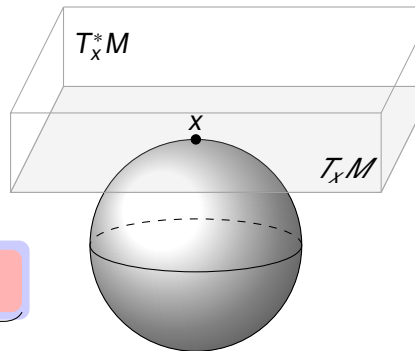
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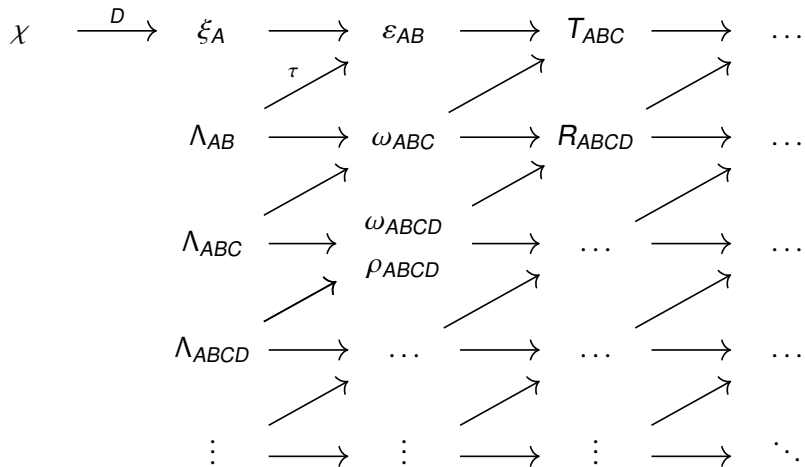
$$\underbrace{\dots \quad \tilde{t}^{ABC} \quad \tilde{t}^{AB}}_{\text{fixed by } \tau^2 = 0 \text{ \& cohomology}} \quad P_A \quad \underbrace{t_{AB} \quad t_{ABC} \quad \dots}_{\text{specified by a symmetric, invariant bilinear form } \kappa \text{ on } \text{Lie}(H)}$$

fixed by $\tau^2 = 0$ & cohomology

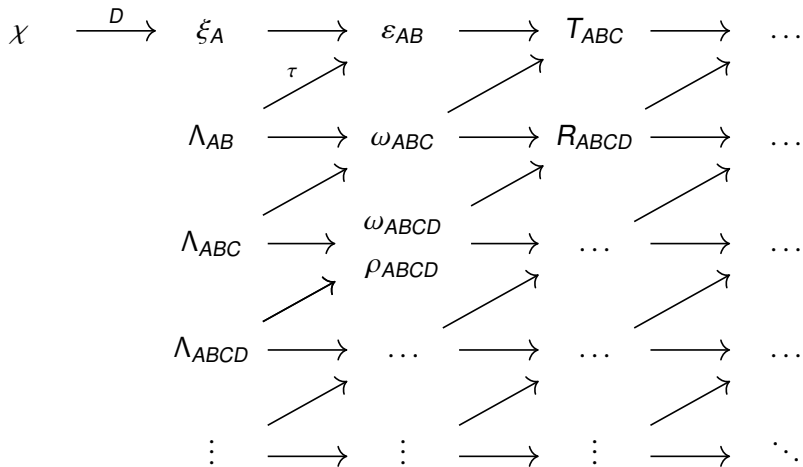
► specified by a symmetric, invariant bilinear form κ on $\text{Lie}(H)$



A tower of corrections



A tower of corrections



$$\Lambda^{i-2} = \Lambda^{A_1 \dots A_i} t_{A_1 \dots A_i}$$

$$\omega_A^{i-2} = \omega_A^{B_1 \dots B_i} t_{B_1 \dots B_i}$$

$\vdots$

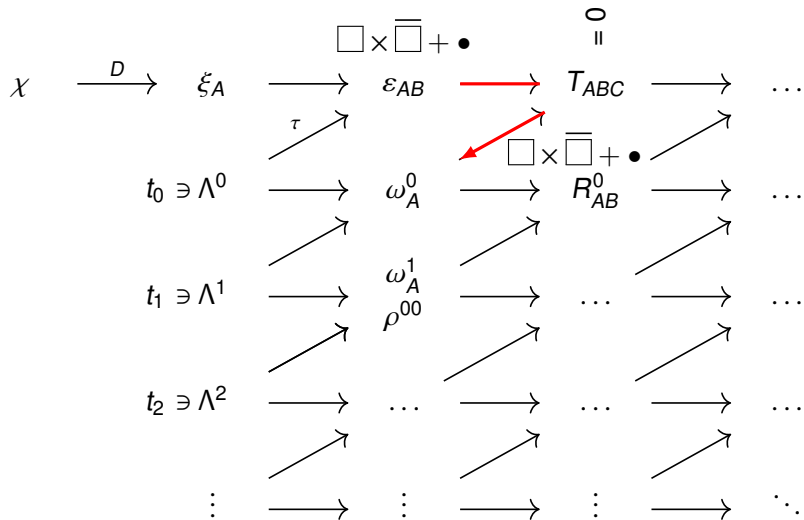
$$\rho^{i-2j-2} = \frac{1}{2} \rho^{A_1 \dots A_i B_1 \dots B_j} t_{A_1 \dots A_i} \wedge t_{B_1 \dots B_j}$$

A tower of corrections

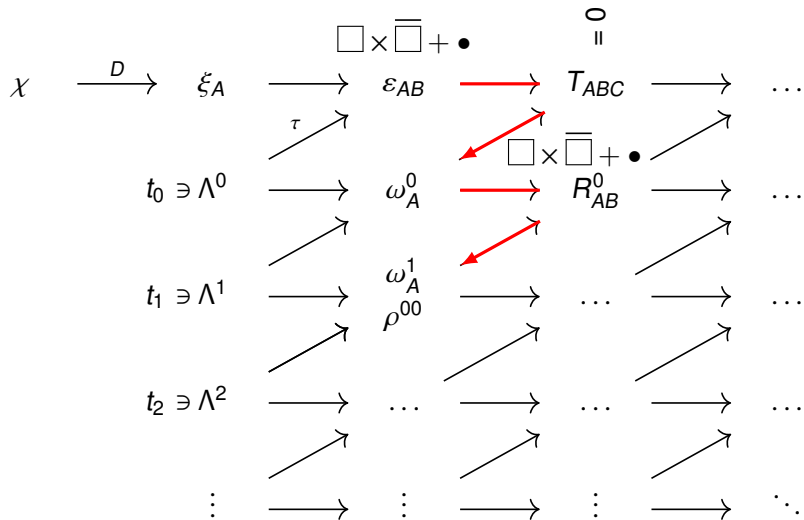
	χ	$\xrightarrow{D}$	ξ_A	$\longrightarrow$	ε_{AB}	$\longrightarrow$	T_{ABC}	$\longrightarrow$	$\dots$
				$\nearrow_{\tau}$		$\nearrow$		$\nearrow$	
			$t_0 \ni \Lambda^0$	$\longrightarrow$	ω_A^0	$\longrightarrow$	R_{AB}^0	$\longrightarrow$	$\dots$
				$\nearrow$		$\nearrow$		$\nearrow$	
$\Lambda^{i-2} = \Lambda^{A_1 \dots A_i} t_{A_1 \dots A_i}$			$t_1 \ni \Lambda^1$	$\longrightarrow$	ω_A^1	$\longrightarrow$	$\dots$	$\longrightarrow$	$\dots$
				$\nearrow$	ρ^{00}	$\nearrow$		$\nearrow$	
$\omega_A^{i-2} = \omega_A^{B_1 \dots B_i} t_{B_1 \dots B_i}$			$t_2 \ni \Lambda^2$	$\longrightarrow$	$\dots$	$\longrightarrow$	$\dots$	$\longrightarrow$	$\dots$
				$\nearrow$		$\nearrow$		$\nearrow$	
$\vdots$			$\vdots$	$\longrightarrow$	$\vdots$	$\longrightarrow$	$\vdots$	$\longrightarrow$	$\ddots$

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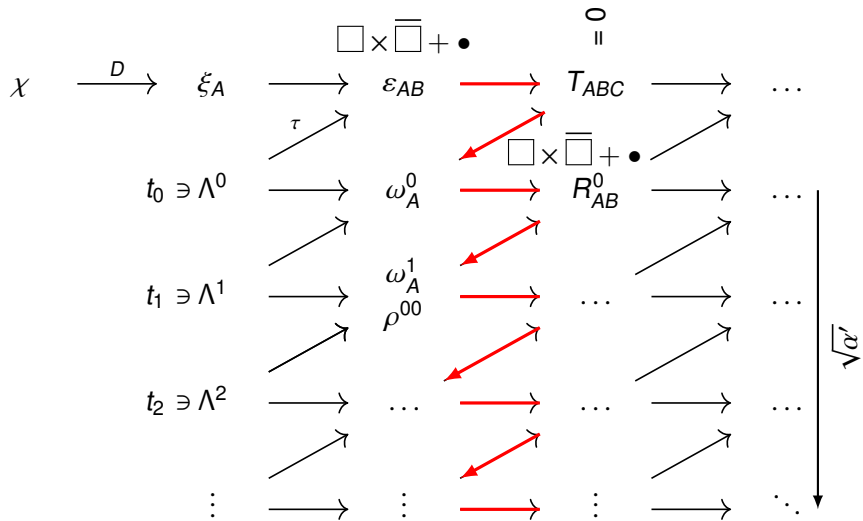
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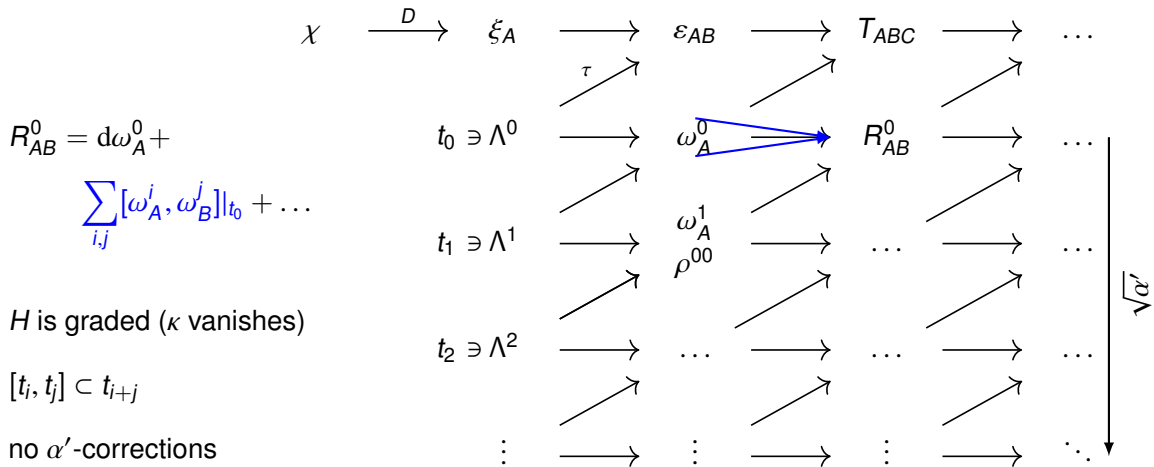
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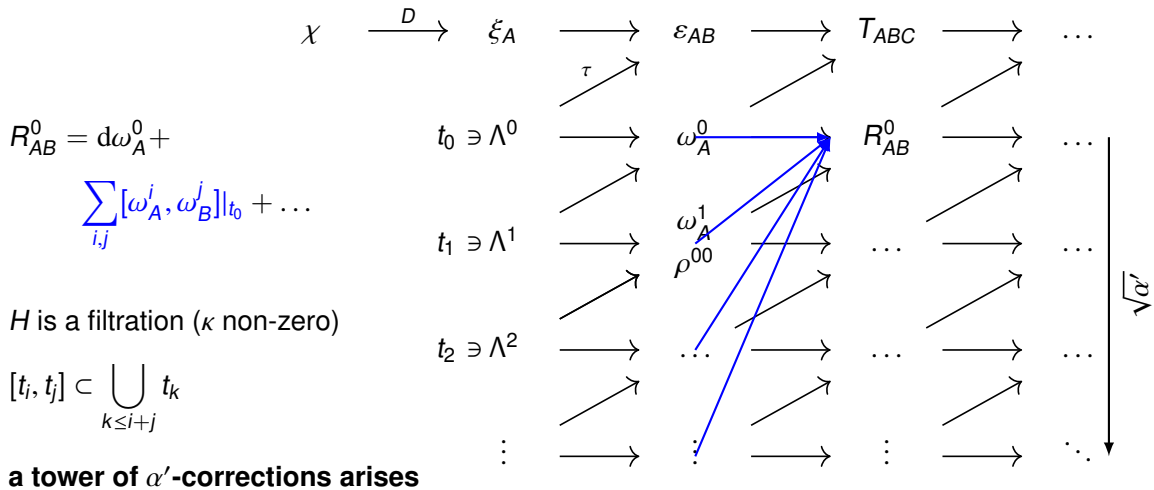
A tower of corrections

$$\begin{array}{ccccccc}
 & \chi & \xrightarrow{D} & \xi_A & \longrightarrow & \varepsilon_{AB} & \longrightarrow & T_{ABC} & \longrightarrow & \dots \\
 & & & \nearrow \tau & & & \nearrow & & \nearrow & \\
 R_{AB}^0 = d\omega_A^0 + & t_0 \ni \Lambda^0 & \longrightarrow & \omega_A^0 & \longrightarrow & R_{AB}^0 & \longrightarrow & \dots & & \\
 \underbrace{\sum_{i,j} [\omega_A^i, \omega_B^j]|_{t_0} + \dots}_{\text{non-linear terms from gen. Cartan}} & t_1 \ni \Lambda^1 & \longrightarrow & \omega_A^1 & \longrightarrow & \dots & \longrightarrow & \dots & & \\
 & \nearrow \rho^{00} & & & \nearrow & & \nearrow & & & \\
 & t_2 \ni \Lambda^2 & \longrightarrow & \dots & \longrightarrow & \dots & \longrightarrow & \dots & & \\
 & \nearrow & & & \nearrow & & \nearrow & & & \\
 & \vdots & \longrightarrow & \vdots & \longrightarrow & \vdots & \longrightarrow & \ddots & & \downarrow \sqrt{\alpha'}
 \end{array}$$

A tower of corrections



A tower of corrections



A skyline and an evaded no-go theorem

- ▶ admissible κ 's are parameterized by $\kappa = \kappa(\underline{a, b}, \underline{c, d}, e, f, \dots)$

very likely (WIP)

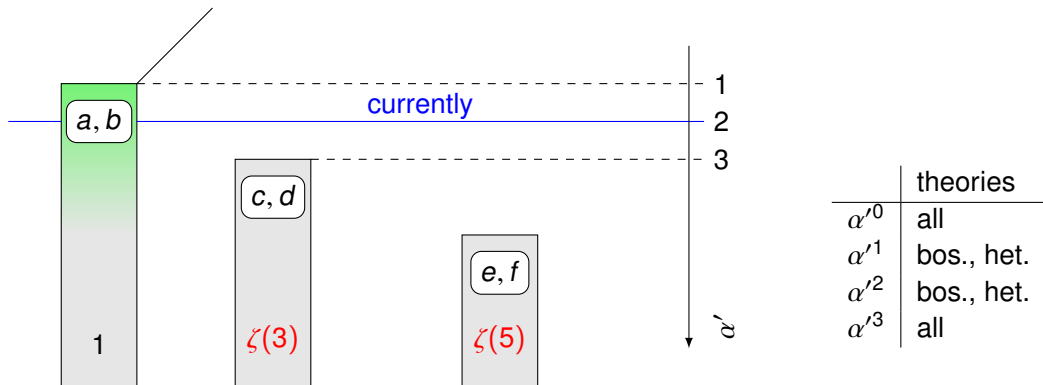
have to be there [Achilleas Gitsis, FH 24]

A skyline and an evaded no-go theorem

- ▶ admissible κ 's are parameterized by $\kappa = \kappa(\underline{a}, \underline{b}, \underline{c}, \underline{d}, e, f, \dots)$
- ▶ each parameter creates a tower of α' -corrections

known from generalized Bergshoeff-de Roo identification

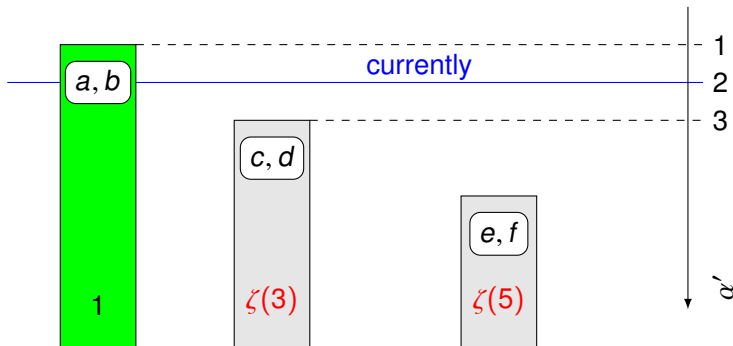
[Bergshoeff, de Roo 89; Marques, Nunez 2015; Baron, Lescano, Marques 2018; Baron, Marques 2022]



A skyline and an evaded no-go theorem

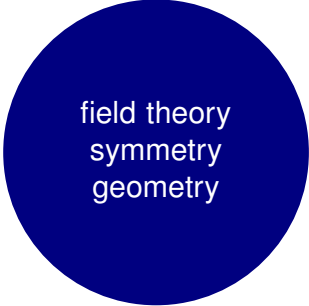
- ▶ admissible κ 's are parameterized by $\kappa = \kappa(\underline{a}, \underline{b}, \underline{c}, \underline{d}, e, f, \dots)$
- ▶ each parameter creates a tower of α' -corrections
- ▶ no-go for α'^3 -tower from deformed $O(d) \times O(d)$ symmetry [Hsia, Kamal, Wulff 24]

No problem, we don't need this symmetry!



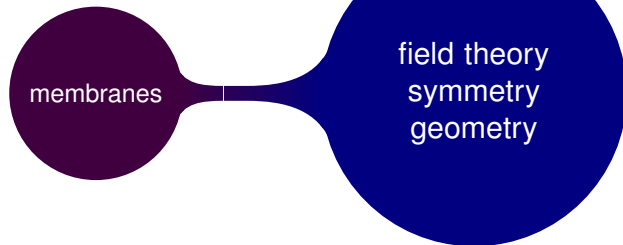
	theories
α'^0	all
α'^1	bos., het.
α'^2	bos., het.
α'^3	all

Where to go from here?



field theory
symmetry
geometry

Where to go from here?

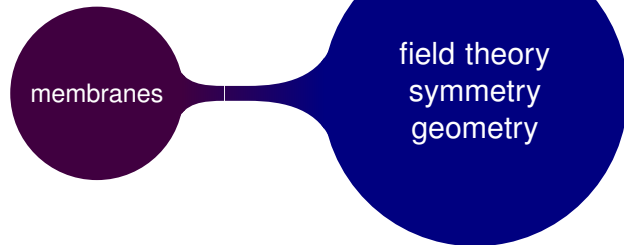


generalized Cartan geometry:

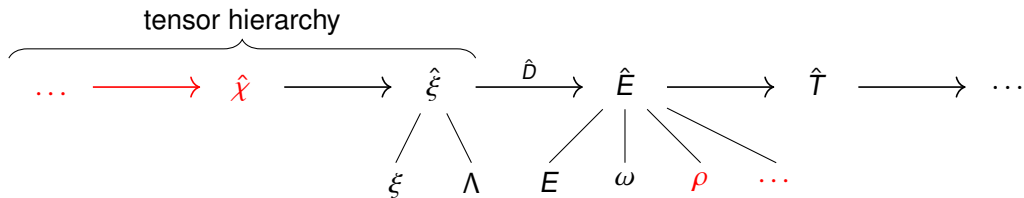
$$\hat{\chi} \longrightarrow \hat{\xi} \xrightarrow{\hat{D}} \hat{E} \longrightarrow \hat{T} \longrightarrow \dots$$

$\xi \quad \Lambda \quad E \quad \omega \quad \rho$

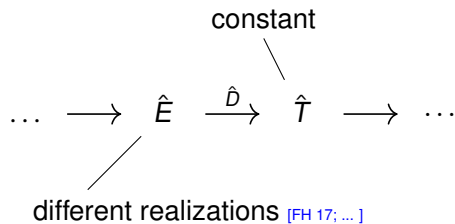
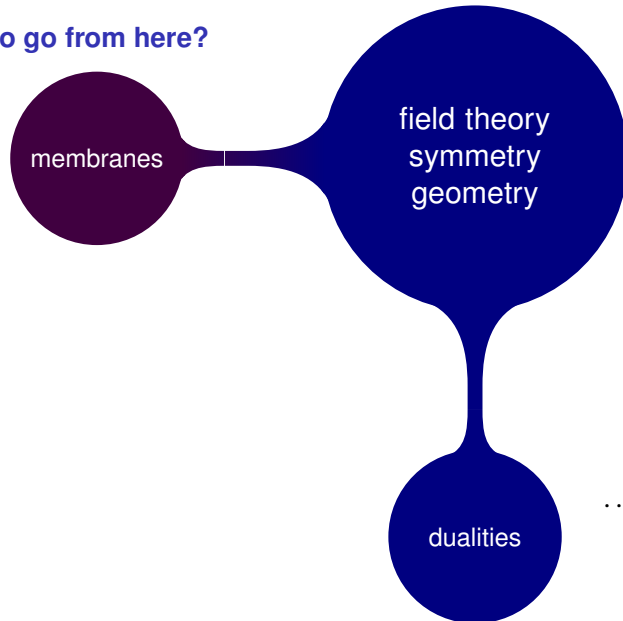
Where to go from here?



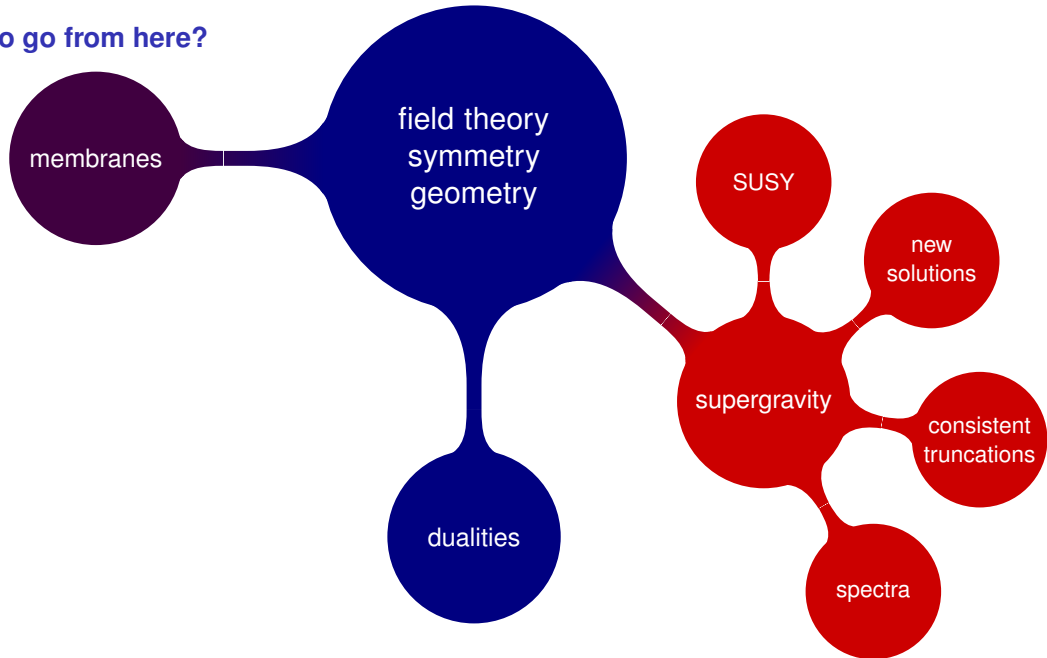
exceptional Cartan geometry: [FH, Yuho Sakatani 23]



Where to go from here?



Where to go from here?



Where to go from here?

