Consistent Truncations & Dualties

Falk Hassler

based on `2211.13241` with Chris Pope, Daniel Butter, Hoayu Zhang

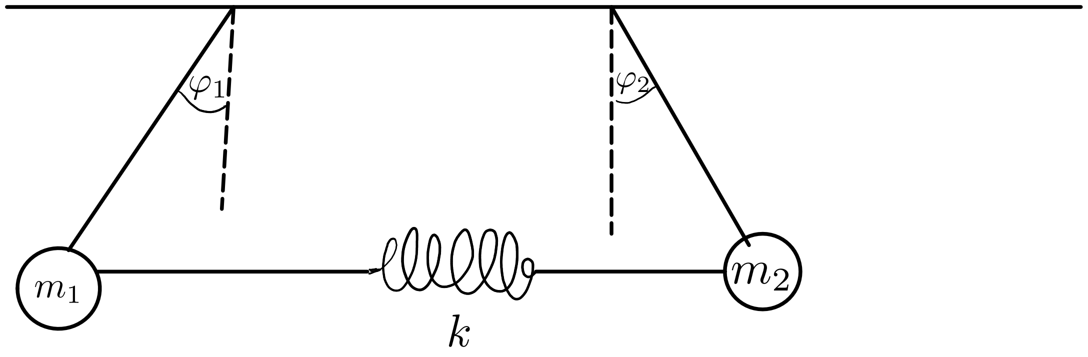
`2212.14886` with Yuho Sakatani

Consistent Truncations

$$\begin{tikzcd}
\text{full action } S \arrow[r, "\text{ansatz}"] \arrow[d] & \text{truncated action } S_{\text{red}} \arrow[d] \\
\text{field equations } \delta S = 0 \arrow[r] & \text{truncated field equations } \delta S_{\text{red}} = 0
\end{tikzcd}$$

$$\underset{\text{uplift}}{\overset{\text{truncation}}{\rightleftarrows}}$$

VERY HARD!!! example



ansatz	consistent ???
$\varphi_2 = 0$	✗
$\varphi_2 = 0\,, m_2 \to \infty$	✓
$\varphi_2 = \varphi_1\,, m_2 = m_1$	✓

more compilcated for supergravity

$$S = \int \mathrm{d}^D e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + \ldots \right)$$

Applications:

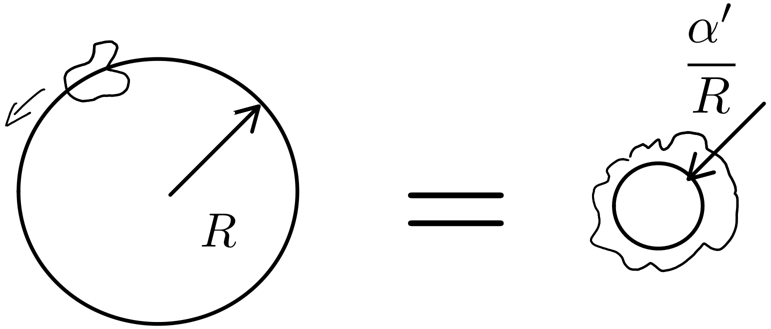
- AdS/CFT
- flux compactifications

  string pheno / swampland
- gauged supergravity

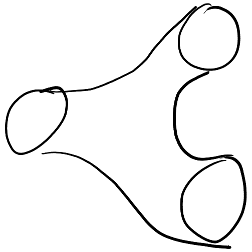
CHALLENGE: find ansätze

Generalised Dualities

T-duality:
arget space



S -duality:
trong/weak



$\sim g_\mathrm{s}$

U-duality = T- Unified with S-duality

requires Abelian isometries (torus)

Just the tip of the iceberg!

$$\begin{array}{ccccccc} \text{Abelian} & \subset & \text{non-Abelian} & \subset & \text{Poisson-Lie} & \subset & \text{WZW-Poisson-Lie}\\ &&&& \cap && \cap \\ &&&& \text{dressing coset} & \subset & \underline{\text{generalised coset}}\end{array}$$

Example: Hamilonian formulation of D-dim. bosonic string

$$H = \frac{1}{4\pi\alpha'} \int \mathrm{d}\sigma J^M \mathcal{H}_{MN} J^N$$

- generalised metric

$$\mathcal{H}_{MN} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}_{MN}$$

and currents $J^M = \begin{pmatrix} \partial_\sigma x^m & p_m \end{pmatrix}$

- canonical momentum

$$p_m = g_{mn}\partial_\tau X^m + B_{mn}\partial_\sigma X^n$$

with Poisson brackets

$$\{J^M(\sigma), J^N(\sigma')\} = 2\pi\alpha'\delta'(\sigma - \sigma')\eta^{MN}$$

- O(D,D) metric

$$\eta^{MN} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}^{MN}$$

IDEA: redefine the current

$$\boxed{J^A = \frac{1}{\sqrt{2\pi\alpha'}} E^A{}_M J^M}$$

generalised fluxes defined by

$$\boxed{\mathcal{L}_{E_A} E_B{}^I = F_{AB}{}^C E_C{}^I}$$

$$\{J^A(\sigma), J^B(\sigma')\} = \delta(\sigma - \sigma') F^{AB}{}_C J^C(\sigma) + \delta'(\sigma - \sigma')\eta^{AB}$$

$$H = \frac{1}{2} \oint \mathrm{d}\sigma J^A \mathcal{H}_{AB} J^B$$

alway possible:

- $\eta^{AB} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}^{AB}$
- $\mathcal{H}_{AB} = \begin{pmatrix} \eta & 0 \\ 0 & \eta^{-1} \end{pmatrix}_{AB}$

Def.: Generalised Lie derivative

$$\mathcal{L}_U V^I = U^J \partial_J V^I - U^J \left(\partial_J V^I \underline{- \partial^I V_J}\right)$$

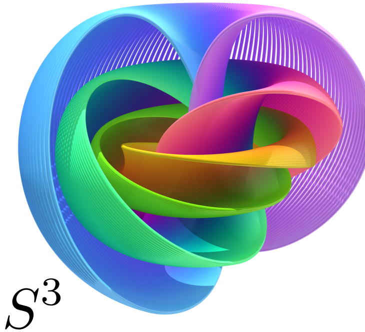
For Abelian ⊂ non-Abelian ⊂ Poisson-Lie ⊂ WZW-Poisson-Lie $F_{ABC} = \text{const.}$

Generalised group manifold (parallelisable space)

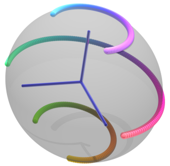
$$L_{e_a} e_b = f_{ab}{}^c e_c{}^i$$

left- or right-invariant vector fields

i.e. on



$\mathrm{SU}(2) = S^3$



① Lie group $\mathcal{D}$ with generators

$$[t_A, t_B] = F_{AB}{}^C t_C$$

② ad-invariant pairing $\langle t_A, t_B \rangle = \eta_{AB}$

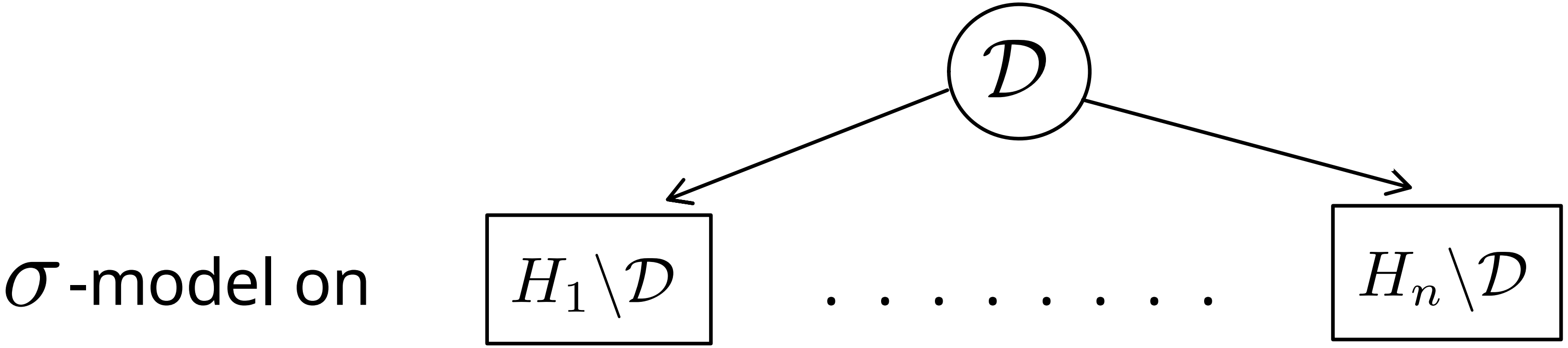
③ maximally isotropic subgroup H

$\langle t^a, t^b \rangle = 0$ generators

$E_A{}^I$

on

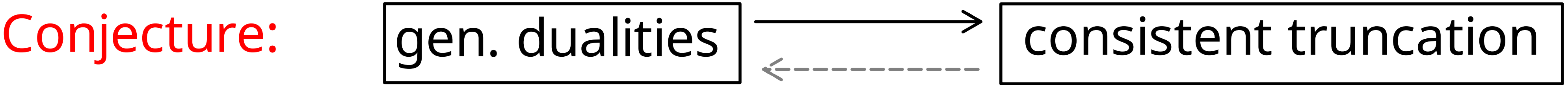
$H \backslash \mathcal{D}$



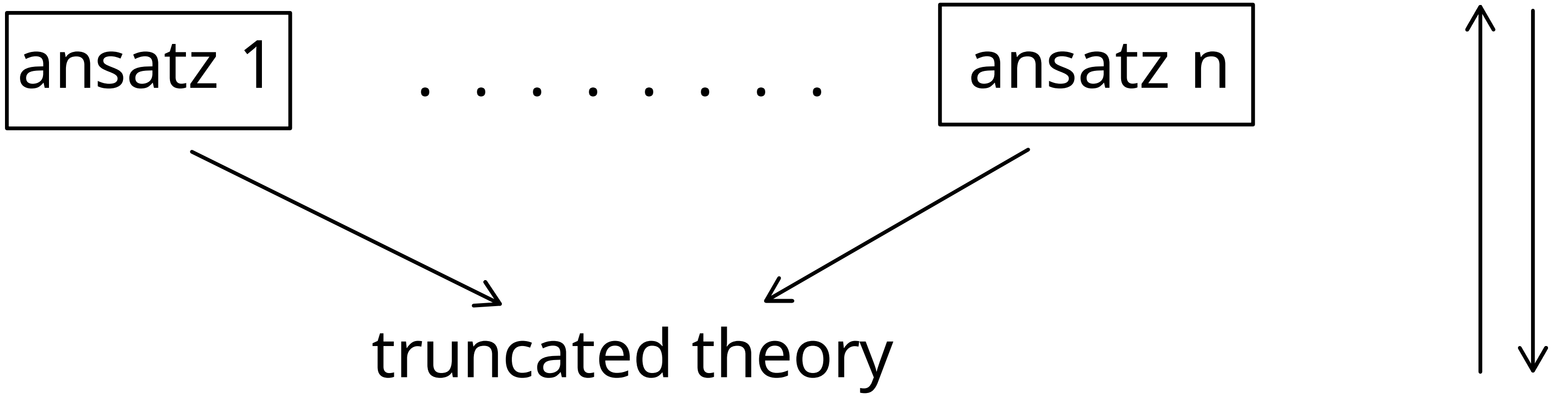
$\mathcal{D}$

σ -model on $H_1\backslash\mathcal{D}$ . . . . $H_n\backslash\mathcal{D}$

Observation: Gen. para. spaces first appeared in consistent truncations



Conjecture: gen. dualities ⇄ consistent truncation



ansatz 1 . . . . ansatz n

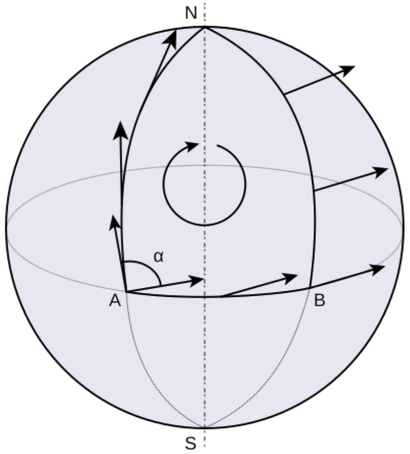
truncated theory

Systematics of consistent truncations

Theorem 1:

[Cassini, Jose,Petrini, Waldram 2019]

Let M be a D-dimensional manifold with generalised F-structure defining a set of invariant tensors and only constant, singlet intrinsic torsion.
Then there is a consistent truncation of supergravity on M by expanding all bosonic fields in terms of the invariant tensors.



covariant derivative:

$$\nabla_I E_A{}^J = \partial_I E_A{}^J - \Omega_{IA}{}^B E_B{}^J + \Gamma_{IK}{}^J E_A{}^K = 0$$

generalised spin connection

Homogeneous spaces (cosets)

Theorem 2: Let (M,g) be a connected and simply-connected complete Riemannian manifold. Then, the following statements are equivalent:

[Ambrose, Singer 1958]

① The manifold M is Riemannian homogenous

② M admits a linear connection ∇ satisfying:

$$\nabla R = 0\,, \quad \nabla S = 0\,, \quad \nabla g = 0$$

Riemann tensor $\quad S = \nabla^{\mathrm{LC}} - \nabla \quad$ meric

Klein geometry: $M = G/\bigcirc\!\!\!\!F$ structure group

Generalised homogeneous spaces (cosets)

$M = H\backslash\mathcal{D}/F$ where F needs to be isotropic

Has covariantly constant:

- generalised torsion
- generalised Riemann tensor
- higher curvature tensor(s)

expected from AS theorem

$$\mathcal{L}_\xi V^I = 0 \quad \forall\, V^I \quad \text{and} \quad \xi^I = \partial^I \chi$$ (gauge for gauge symmetry)

May contain singularites!

Example: SL(2)/U(1) gauged WZW model (cigar and T-dual trumpet)

Remarks

- covers all known generalised T-dualities

$$\begin{array}{ccccccc} \text{Abelian} & \subset & \text{non-Abelian} & \subset & \text{Poisson-Lie} & \subset & \text{WZW-Poisson-Lie} \\ &&&& \cap && \cap \\ &&&& \text{dressing coset} & \subset & \underline{\text{generalised coset}} \end{array}$$

- many new consistent truncations
- covers all known integrable σ-models
- generalisations to
  - supergroups (done)

    full type II with fermions
  - heterotic/type I (nearly done)
  - U-dualities (work in progress)

Prospective

Observation: The underlying structures (i.e. Poisson-Lie groups) are the classical limit of quantum groups and quantum homogenous spaces.

$$\{\cdot\,,\,\cdot\} \to -i\hbar[\cdot\,,\,\cdot]$$

to obtain higher derivative corrections

and explore applications to

- integrability
- holography
- flux compactifications
- ...