Tutorial for String Theory I, WiSe2013/14
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13. D-branes

To be discussed on Thursday, January 30, 2013 in the tutorial.

Exercise 13.1: D-brane mode expansion

Let x^μ ($\mu = 0, \dots, p$) denote the spacetime coordinates tangential to a Dp-brane and use x^m ($m = p + 1, \dots, 25$) for the transverse directions.

a) What kind of boundary conditions (Neumann or Dirichlet) does one have to impose on $X^\mu(\sigma, \tau)$ and on $X^m(\sigma, \tau)$ if the string begins and ends on the Dp-brane?

b) What kind of boundary conditions ((N) or (D)) does one have to impose on $X^\mu(\sigma, \tau)$ and on $X^m(\sigma, \tau)$ if the string begins on the Dp-brane but has the other end moving freely in spacetime?

c) For a string that begins and ends on a Dp-brane at position $\bar{x}^a$, we have, for the transverse coordinates $X^a(\sigma, \tau)$,

$$X^a(\tau, 0) = X^a(\tau, \pi) = \bar{x}^a \,.$$

Furthermore, the fact that X^a solves the $2D$ wave equation implies that

$$X^a(\sigma, \tau) = \frac{1}{2}\left(f^a(\tau + \sigma) + g^a(\tau - \sigma)\right) \tag{1}$$

for some as yet arbitrary functions f^a and g^a. Evaluate (1) at $\sigma = 0$ to show that

$$X^a(\sigma, \tau) = \bar{x}^a + \frac{1}{2}\left(f^a(\tau + \sigma) - f^a(\tau - \sigma)\right) \,.$$

d) Use the boundary condition at $\sigma = \pi$ to derive

$$f^a(\tau + \pi) = f^a(\tau - \pi) \,.$$

e) The result of part d) means that f^a is a periodic function of its argument with period 2π. Show that this forbids a linear term in τ in $X^a(\tau, \sigma)$. What is the physical significance of the absence of a linear term in τ in the mode expansion of $X^a(\tau, \sigma)$?

f) Does one have a linear term in τ in $X^\mu(\tau, \sigma)$? Putting everything together, what is the physical consequence of the observation in parts e) and f) for the open string states with both ends on a Dp-brane?

Exercise 13.2: Strings between D-branes with different dimensionality

Consider a string stretching between a Dp-brane and a parallel Dq-brane with $1 \leq q < p \leq 25$.

a) Assume $p = 2$ and $q = 1$ to draw a figure illustrating the situation.

b) Classify the string coordinates X^μ according to the boundary conditions they have to fulfill.

c) Write down the boundary conditions of the coordinates X^r, $r = q+1, \ldots, p$

Hint: These are the coordinates which have to fulfill mixed Neumann-Dirichlet boundary conditions.

d) Show that the mode expansion of X^r, $r = q+1, \ldots, p$ can be written as

$$X^r(\tau, \sigma) = \bar{x}^r + \sum_{n \in \mathbb{Z}^+_{\text{odd}}} \left(A^r_n \cos\left(\frac{n\tau}{2}\right) + B^r_n \sin\left(\frac{n\tau}{2}\right) \right) \cos\left(\frac{n\sigma}{2}\right) ,$$

where $\bar{x}^r$ is the position of the Dq-brane in the r-directions, and A^r_n, B^r_n are expansion coefficients.