O(D,D)-covariant β-functions

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based on

2011.15130 with Thomas Rochais and

2012.10451

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Outline

1. **Revealing a new symmetry**
2. **Physical interpretation: abelian & generalised T-duality**
3. **One- & two-loop RG flow**
4. **Open questions**

2-dimensional σ-model, the Swiss army knife of field theories

$$\boxed{S_\Sigma = \frac{1}{4\pi\alpha'}\int_\Sigma \left(g_{ij}\mathrm{d}x^i \wedge \star\mathrm{d}x^j + B_{ij}\mathrm{d}x^i \wedge \mathrm{d}x^j + \star\phi R\right)}$$

- D bosons x^i, $i = 1, \ldots, D$ coupled to gravity (topological)
- couplings $g_{ij}, \ldots$ describe the geometry of a Target space M
- fields $x^i(\tau, \sigma)$ embed the worldsheet Σ into M
- related to other (¿all?) QFTs by string theory

Discovering a new symmetry

1. S_Σ in Hamiltonian formalism: [Tseytlin, 1990]

$$S_\Sigma = \int_\Sigma \mathrm{d}\sigma \mathrm{d}\tau \dot{x}^i p_i - \int \mathrm{d}\tau H(\tau)$$

$$H(\tau) = \frac{1}{4\pi\alpha'} \int \mathrm{d}\sigma J_M \mathcal{H}^{MN} J_N$$

with Poisson bracket $\{J_M(\sigma), J_N(\sigma')\} = 2\pi\alpha' \eta_{MN} \delta'(\sigma - \sigma')$

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2. Make $H(\tau)$ as simple as possible

$$\mathcal{H}^{MN} = E_A{}^M E_B{}^N \mathcal{H}^{AB} \qquad \mathcal{H}^{AB} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

$$J_M = \frac{1}{\sqrt{2\pi\alpha'}} E^A{}_M J_A \qquad \delta_B^A = E^A{}_M E_B{}^M$$

but more complicated Poisson bracket [Siegel, 1993]

$$\{J_A(\sigma), J_B(\sigma')\} = F_{AB}{}^C J_C(\sigma) \delta(\sigma - \sigma') + \eta_{AB} \delta'(\sigma - \sigma')$$

$\mathcal{E}$-model and Poisson-Lie symmetry [Klimčík and Ševera, 1995;Klimčík and Ševera, 1996]

relevant quantities: $\eta_{AB} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$, $\mathcal{H}_{AB} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$, and $F_{AB}{}^C$

$$F_{ABC} = F_{AB}{}^D \eta_{DC} = 3 E_{[A}{}^I \partial_I E_B{}^J E_{C]J} \text{ with } \partial_I = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$$

encodes all σ-model data (g_{ij} and B_{ij}) in one quantity

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PL symmetry: $F_{AB}{}^C$ are structure constants of Lie algebra $\mathfrak{d}$

1. $[T_A, T_B] = F_{AB}{}^C T_C \qquad T_A \in \mathfrak{d}$
2. ad-invariant pairing $\langle T_A, T_B \rangle = \eta_{AB}$
3. involution $\mathcal{E}^A{}_B := \eta^{AC} \mathcal{H}_{CB}$ with $\mathcal{E}^2 = 1$ and $\langle \mathcal{E} T_A, T_B \rangle = \langle T_A, \mathcal{E} T_B \rangle$

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$\mathcal{E}$-model

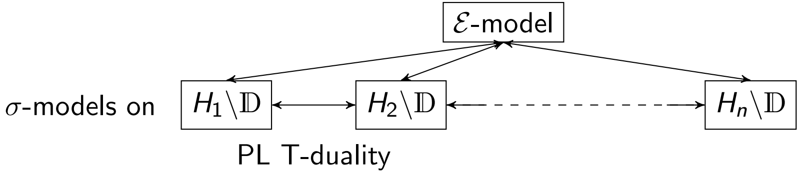
$$H = \frac{1}{2} \int \mathrm{d}\sigma \, J_A \mathcal{H}^{AB} J_B$$

$$\{J_A(\sigma), J_B(\sigma')\} = F_{AB}{}^C J_C(\sigma) \delta(\sigma - \sigma') + \eta_{AB} \delta'(\sigma - \sigma')$$

T-duality and integrable deformations

- for a fixed $\mathfrak{d}$ there is a different ${E_A}^I$ for each

  max. isotropic subalgebra $\mathfrak{h} \ni T^a$, $a = 1, \dots, D$, with $\langle T^a, T^b \rangle = 0$



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$$
\begin{tikzcd}
 & & & \boxed{\mathcal{E}\text{-model}} \arrow[dll, <->] \arrow[dl, <->] \arrow[dr, <->] & \\
\sigma\text{-models on} & \boxed{H_1 \backslash \mathbb{D}} \arrow[r, <->, "\text{PL T-duality}"'] & \boxed{H_2 \backslash \mathbb{D}} \arrow[rr, <->, dashed] & & \boxed{H_n \backslash \mathbb{D}}
\end{tikzcd}
$$

- $\mathcal{E}$-model field equations:

$$\dot{J}_A = \{J_A, H\} \qquad \leftrightarrow \qquad \mathrm{d}J + \frac{1}{2}[J, J] = 0$$

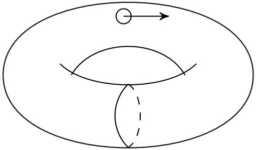
$$J = T_A \left(\mathcal{E}^A{}_B J^B \mathrm{d}\tau + J^A \mathrm{d}\sigma \right)$$

similar to flat Lax connection

for integrable $\mathcal{E}$-models map between J and $\mathcal{L}$ [Ševera, 2017]

Abelian T-duality

- target space with abelian isometries



winding $\leftrightarrow$ momentum

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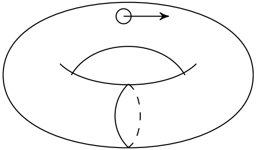


winding $\leftrightarrow$ momentum

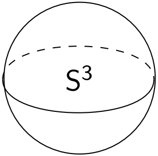
- on the worldsheet Buscher procedure [Buscher 87]

1. gauge global U(1) symmetry
2. use Lagrange multiplier λ to impose $F = \mathrm{d}A = 0$
3. integrate out λ or A

original model $\qquad\qquad$ dual model

4. Wilson loops $\oint A$ fix periodicity of λ

Non abelian T-duality [de la Ossa, Quevedo 93]



- idea: gauge non-abelian symmetry on worldsheet
- problem: λ now in the adjoint & not a singlet

  1. global properties of dual model do not arise as in the abelian case
  2. isometry group of the dual target space is smaller

$\downarrow$

¿NATD is not invertible?

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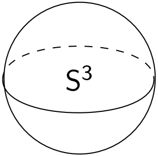

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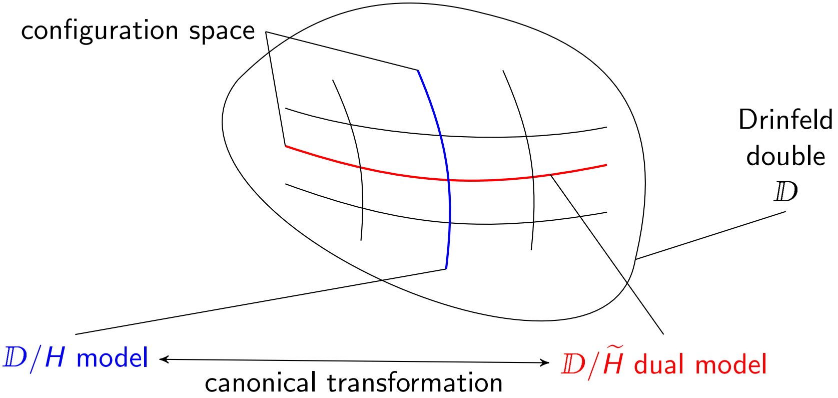
¿NATD is not invertible?

[Giveon, Roček 93]

We argue that, except for "accidents", there is no reason to expect nonabelian duality to be a symmetry of a CFT; at best, it can be a transformation between different CFT's.

Poisson-Lie T-duality [Klimčík, Ševera 95]

NATD is invertible & we should look at the phase space



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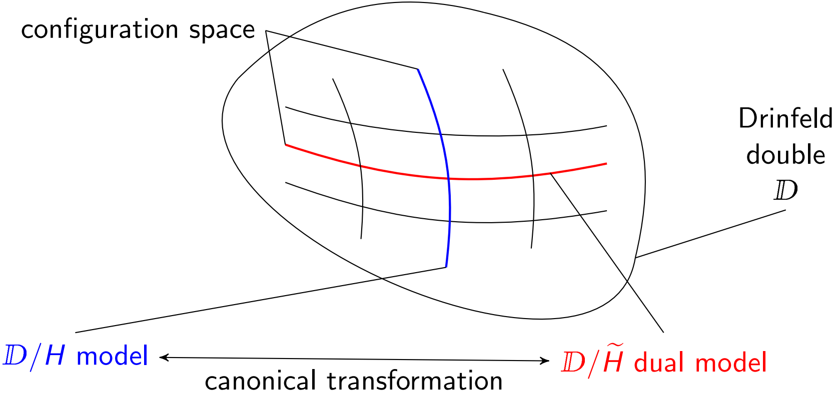

$\mathbb{D}$ = Lie group with two max. iso. subgroups; $\mathbb{D} = H \ltimes \widetilde{H}$

H and $\widetilde{H}$ are Poisson-Lie groups

Generalised T-duality

dressing coset construction — double coset $F \backslash \mathbb{D} / H$ [Klimčík, Ševera 96]; most general; includes $\mathrm{AdS}_5 \times \mathrm{S}^5$

$\cup$

Poisson-Lie + WZW — $\mathbb{D}$ with <u>one</u> max. iso. subgroup

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Poisson-Lie — H and $\widetilde{H}$ are non-abelian

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non-abelian — H or $\widetilde{H}$ is abelian

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abelian — H and $\widetilde{H}$ are abelian

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classical! quantum corrections?

PL symmetry & Double Field Theory [FH 17, Çatal-Özer, Demulder, Sakatani, Thompson, ...]

Remember: Poisson-Lie symmetric σ-model ($\mathcal{E}$-model)

- 2 D-dimensional Lie algebra $\mathfrak{d}$ with $T_A \in \mathfrak{d}$

  $[T_A, T_B] = F_{AB}{}^C T_C$

- ad-invariant O(D,D) pairing

  $\langle T_A, T_B \rangle = \langle T_B, T_A \rangle = \eta_{AB}$

- involution $\mathcal{E} : \mathfrak{d} \to \mathfrak{d}$, $\mathcal{E}^2 = 1$

  $\langle T_A, \mathcal{E} T_B \rangle = \langle \mathcal{E} T_A, T_B \rangle = \mathcal{H}_{AB}$

⋆ element from the center of $\mathfrak{d}$

  $F^A t_A \,, \qquad F_{AB}{}^C F_C = 0$

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¿How to get the metric g, the B-field B and the dilaton ϕ?

Generalised frame fields

- $E^A{}_I(x)$ with $E^A{}_I \eta_{AB} E^B{}_J = \begin{pmatrix} 0 & \delta_j^i \\ \delta_i^j & 0 \end{pmatrix}$

such that $\boxed{\begin{aligned} \mathcal{L}_{E_A} E_B &= F_{AB}{}^C E_C \\ \mathcal{L}_{E_A} e^{-2d} &= -F_A e^{-2d} \end{aligned}}$ holds (frame algebra)

generalised Lie derivative

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- $\mathcal{L}_\xi V^I = \xi^J \partial_J V^I + V_J \partial^I \xi^J - V^J \partial_J \xi^I, \quad \partial_I = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$

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- g, B and ϕ are encoded in

  gen. dilaton $d = \phi - \frac{1}{4} \log \det g$

  gen. metric $\mathcal{H}_{IJ} = \begin{pmatrix} g^{ij} & g^{ik} B_{kj} \\ -B_{ik} g^{kj} & g_{ij} - B_{ik} g^{kl} B_{lj} \end{pmatrix} = E^A{}_I \mathcal{H}_{AB} E^B{}_J$

RG flow

- the σ-model is a 2-dim. QFT with ∞ number of couplings
- How do they flow from the UV $\to$ IR?

$$\frac{\mathrm{d}g_{ij}}{\mathrm{d}t} = R_{ij} \qquad t = \log\mu \qquad \text{(one-loop)}$$

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- with B-field and dilaton generalised Ricci flow

$$\frac{\mathrm{d}\mathcal{H}_{IJ}}{\mathrm{d}t} = \mathcal{R}_{IJ} \quad \text{—— generalised Ricci tensor [Hohm, Hull, Zwiebach 10]}$$

PL symmetry & one-loop RG flow

- go to adapted frame: $\mathcal{H}_{IJ} \xrightarrow{E_A{}^I} \mathcal{H}_{AB}$
- restrictions on $\dot{\mathcal{H}}_{AB} = \langle T_A, \dot{\mathcal{E}} T_B \rangle = \mathcal{R}_{AB}$

$$\mathcal{E}^2 = 1 \qquad \rightarrow \qquad 0 = P\dot{\mathcal{E}}P = \overline{P}\dot{\mathcal{E}}\overline{P} \qquad\qquad P = \frac{1}{2}(1+\mathcal{E})$$

$$\overline{P} = \frac{1}{2}(1-\mathcal{E})$$

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$$\mathcal{R}_{AB} = \begin{pmatrix} 0 & \mathcal{R}_{a\bar{b}} \\ \mathcal{R}_{\bar{a}b} & 0 \end{pmatrix}$$

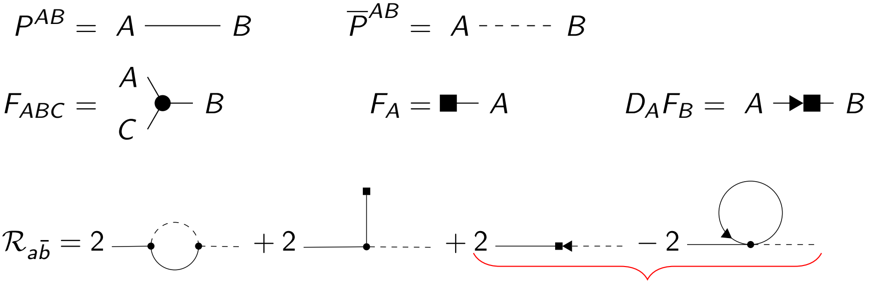
encodes one-loop RG flow

$$P^A{}_B = \begin{pmatrix} \delta^a_b & 0 \\ 0 & 0 \end{pmatrix} \qquad \overline{P}^A{}_B = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\bar{a}}_{\bar{b}} \end{pmatrix}$$

“Feynman”-diagrams

$$\mathcal{R}_{a\bar{b}} = 2P_a{}^C\overline{P}_{\bar{b}}{}^D\left(F_{CEG}F_{DFH}P^{EF}\overline{P}^{GH} + F_{CDE}F_E P^{EF} + D_D F_C - D_E F_{CDF}\overline{P}^{EF}\right)$$

with $D_A = E_A{}^I\partial_I$ and $\partial_I = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$ [Geissbuhler, Marqués, Nuñez, Penas 13]



= DFT field equations

killed by PL symmetry

Challenges beyond one-loop I

β-functions are scheme dependent!

scheme = coordinate choice on coupling space

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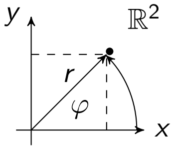

diffeomorphism
$x = r\cos\varphi$
$y = r\sin\varphi$

an infinitesimal coordinate change
$x^\mu \to x^\mu - \xi^\mu$
changes a vector v^μ
$v^\mu \to v^\mu + L_\xi v^\mu$

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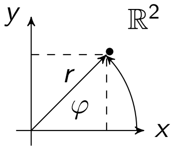

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$$x^\mu \sim \begin{pmatrix} g_{ij} & B_{ij} & \phi \end{pmatrix}$$

$$\xi^\mu \sim \begin{pmatrix} \Delta g_{ij} & \Delta B_{ij} & \Delta\phi \end{pmatrix} = \Psi$$

$$v^\mu \sim \begin{pmatrix} \beta^g_{ij} & \beta^B_{ij} & \beta^\phi \end{pmatrix} = \beta$$

$$\xi^\mu \partial_\mu = \delta_\Psi = \Delta g_{ij} \frac{\delta}{\delta g_{ij}} + \dots$$

$$\beta \to \beta + L_\Psi \beta$$

$$\boxed{L_\Psi = \delta_\Psi \beta - \delta_\beta \Psi - T(\Psi, \beta)}$$

δ can have torsion

The "right" scheme [Marqués, Nuñez 15]

- observables are scheme independent

  BUT action of symmetries depend on the scheme

- keep frame algebra unmodified $\boxed{\mathcal{L}_{E_A} E_B = F_{AB}{}^C E_C}$

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$\rightarrow$ generalised Bergshoeff-de Roe scheme (Marqués-Nuñez scheme)

↯ double Lorentz transformations are modified

$\rightarrow$ keep track of $E_A{}^I$'s double Lorentz frame

frame algebra $E_A{}^I$ $\longleftrightarrow$ gen. BdR scheme $\widehat{E}_A{}^I$

finite generalised Green-Schwarz transformation
generate α'-correction for metric, B-field & dilaton

[Borsato, López, Wulff 20; FH, Rochais 20; Borsato, Wulff 20; Codina, Marqués 20]

Challenges beyond one-loop II

- in right scheme two-loop field equations exclusively depend on

  [Baron, Fernández-Melgarejo, Marqués, Nuñez 17]

$$F_{ABC}\,, \qquad F_A\,, \qquad P^{AB}\,, \qquad \overline{P}^{AB}\,, \qquad \text{and} \qquad D_A$$

↯ BUT field equations $\neq \beta$-functions

instead $\delta_\Psi S = \int \mathrm{d}^D x e^{-2d} \Psi \cdot K(\beta)$

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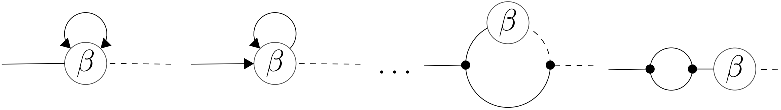
¿Is it possible to write $K^{(2)}$ just with $\bullet$?

Sketch of the computation

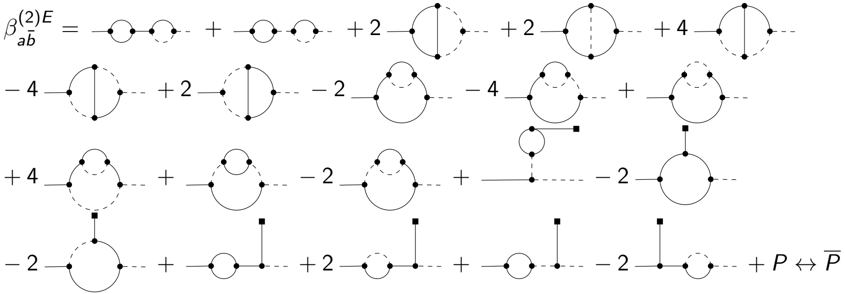
- obtain $K^{(2)}$ in the Metsaev-Tseytlin scheme
- transform it to the gen. BdR scheme
- write result in terms of H-flux H_{abc}, spin connection $\omega_{ab}{}^c$ and F_a
- match 77 terms with 19 doubled diagrams like



- works despite 4:1 overdetermined

Two-loop β-function

- 342 diagrams vs. 4 @ one-loop
- transform covariantly under gen. Green-Schwarz transformations
- imposing PL symmetry and → 40 diagrams remain



PL symmetry is preserved under two-loop RG flows!

Open questions

- is there a "geometric" interpretation, like for $\mathcal{R}_{AB}$, for $\beta^{(2)}_{AB}$
- obtaining β-functions directly from the $\mathcal{E}$-model
- can dressing cosets be treated in the same way
- two-loop heterotic string should be manageable
- explore the integrability/RG flow correspondence further
- how do quantum groups fit into the picture
- is it possible to study irrelevant deformations, like $T\overline{T}$
- operator map under PL T-duality
- ...