Poisson-Lie Symmetry and Double Field Theory

Part I

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based on

1810.11446,
1707.08624, 1611.07978,
1502.02428, 1410.6374

and work in progress

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Motivation: I) $\mathcal{E}$-Model ...

$$S = S_{\mathrm{WZW}} - \frac{1}{2}\int \langle l^{-1}\partial_\sigma l, \mathcal{E} l^{-1}\partial_\sigma l\rangle$$

$$S_{\mathrm{WZW}} = \frac{1}{2}\int \mathrm{d}\sigma\,\mathrm{d}\tau \langle l^{-1}\partial_\sigma l, l^{-1}\partial_\tau l\rangle + \frac{1}{12}\int \langle [l^{-1}\mathrm{d}l, l^{-1}\mathrm{d}l], l^{-1}\mathrm{d}l\rangle$$

- target space Lie group $\mathcal{D} \ni l$ with maximal isotropic subgroup $\tilde{G}$
- Poisson-Lie symmetry and T-duality are manifest symmetries
- integrate out 1/2 of the degrees of freedom $\rightarrow$ σ-model on $\mathcal{D}/\tilde{G}$
- $\mathcal{D}$ captures the phase space of this σ-model

$$J = T_A J^A = l^{-1}\partial_\sigma l$$

$$H = \frac{1}{2}\int \mathrm{d}\sigma \langle J, \mathcal{E} J\rangle$$

$$\{J^A(\sigma), J^B(\sigma')\} = F^{AB}{}_C J^C(\sigma)\delta(\sigma - \sigma') + \eta^{AB}\partial_\sigma\delta(\sigma - \sigma')$$

Motivation: ... and integrable deformations of the PCM

- field equations from $\partial_\tau J^A = \{H, J^A\}$
- example principle chiral model (PCM) $J = j_0 + j_1$

$$\begin{aligned} \partial_\tau j_0 - \partial_\sigma j_1 &= 0 \\ \partial_\tau j_1 - \partial_\sigma j_0 - [j_0, j_1] &= 0 \end{aligned}$$

- Zakharov-Mikhailov field equations $\rightarrow$ Lax pair
- Lax pair $\rightarrow$ infinite number of conserved charges $\rightarrow$ integrable
- new integrable model
  1. deform $\{\ ,\ \}$ and keep H
  2. such that field equations do not change

Because both $\{\ ,\ \}$ and H are manifest in the $\mathcal{E}$-model it is perfectly suited to explore these deformations.

¿Low energy effective target space theory?

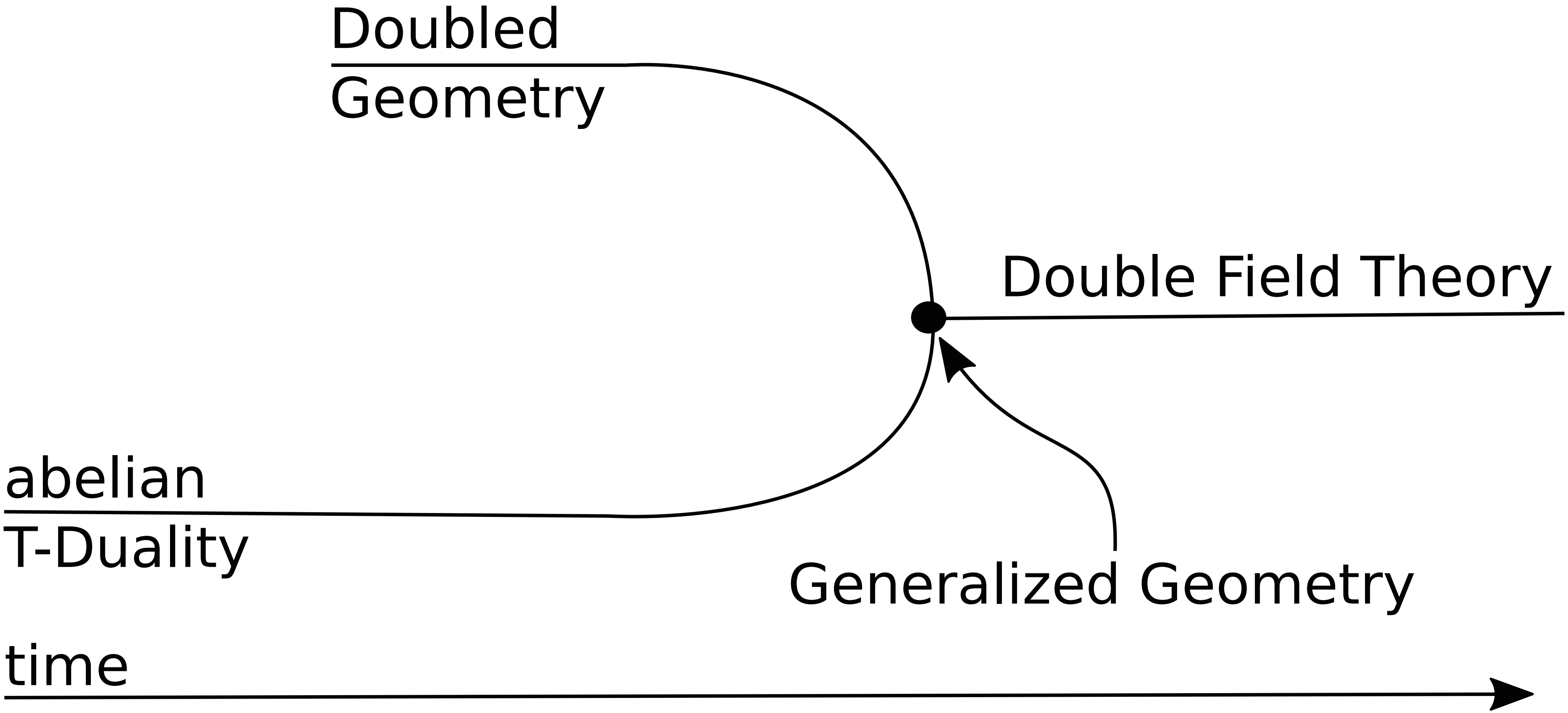
- $\mathcal{E}$-model = σ-model on $\mathcal{D}/\tilde{G} \rightarrow$ (g)SUGRA
- but then we loose all the nice structure on $\mathcal{D}$
- $\mathcal{E}$-model = doubled σ-model $\rightarrow$ Double Field Theory?

  CHALLENGES
  1. doubled space = winding + normal coordinates $\neq \mathcal{D}$
  2. abelian T-duality is manifest $\subset$ Poisson-Lie T-duality

$\rightarrow$ standard DFT does not work

Today, I will show you how to change the standard DFT framework to overcome these challenges. The result is called DFT on group manifolds (abbreviated DFT_{WZW}) and will meet all our expectations.

Motivation: II) Incorporate Poisson-Lie T-duality into DFT



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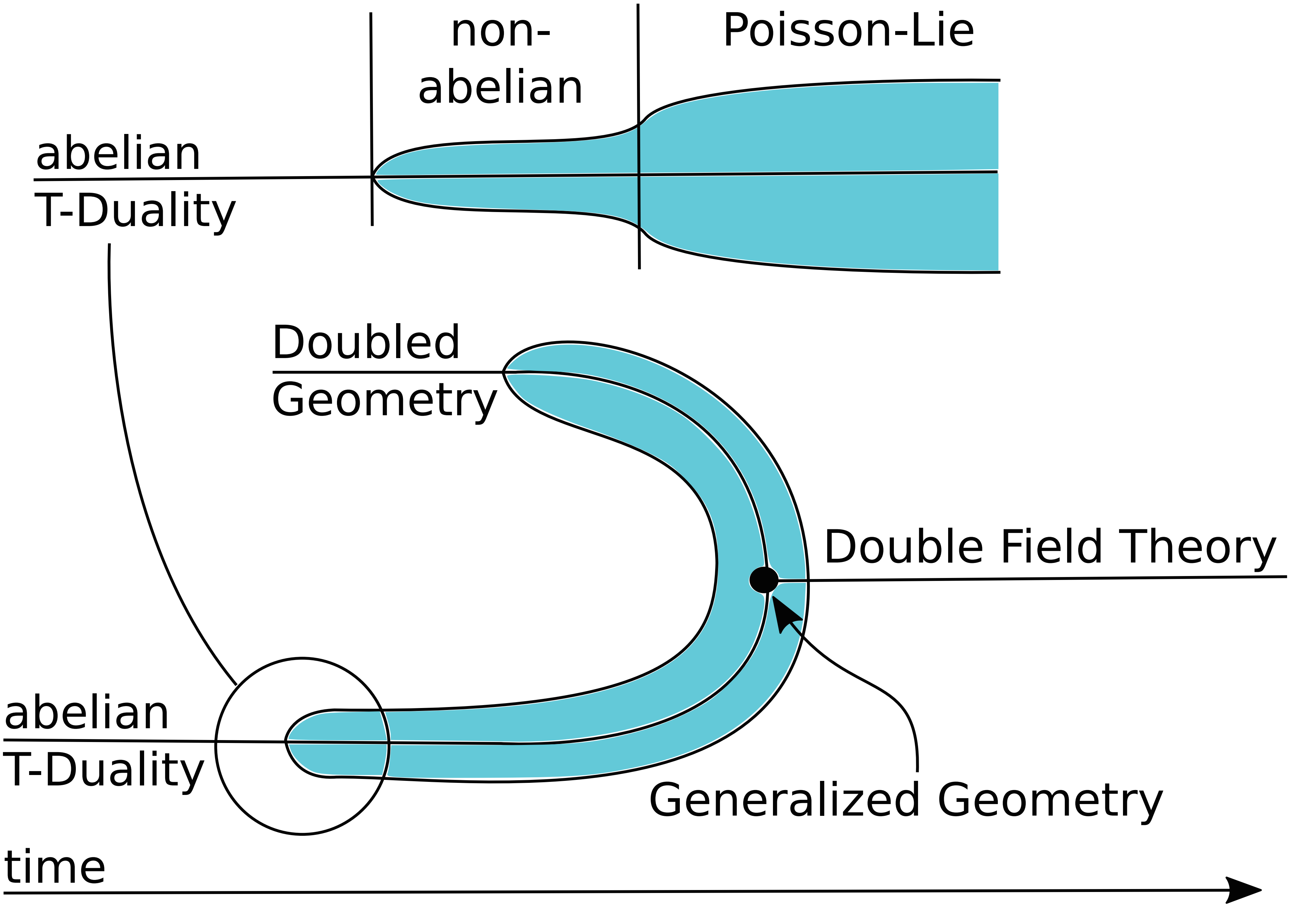

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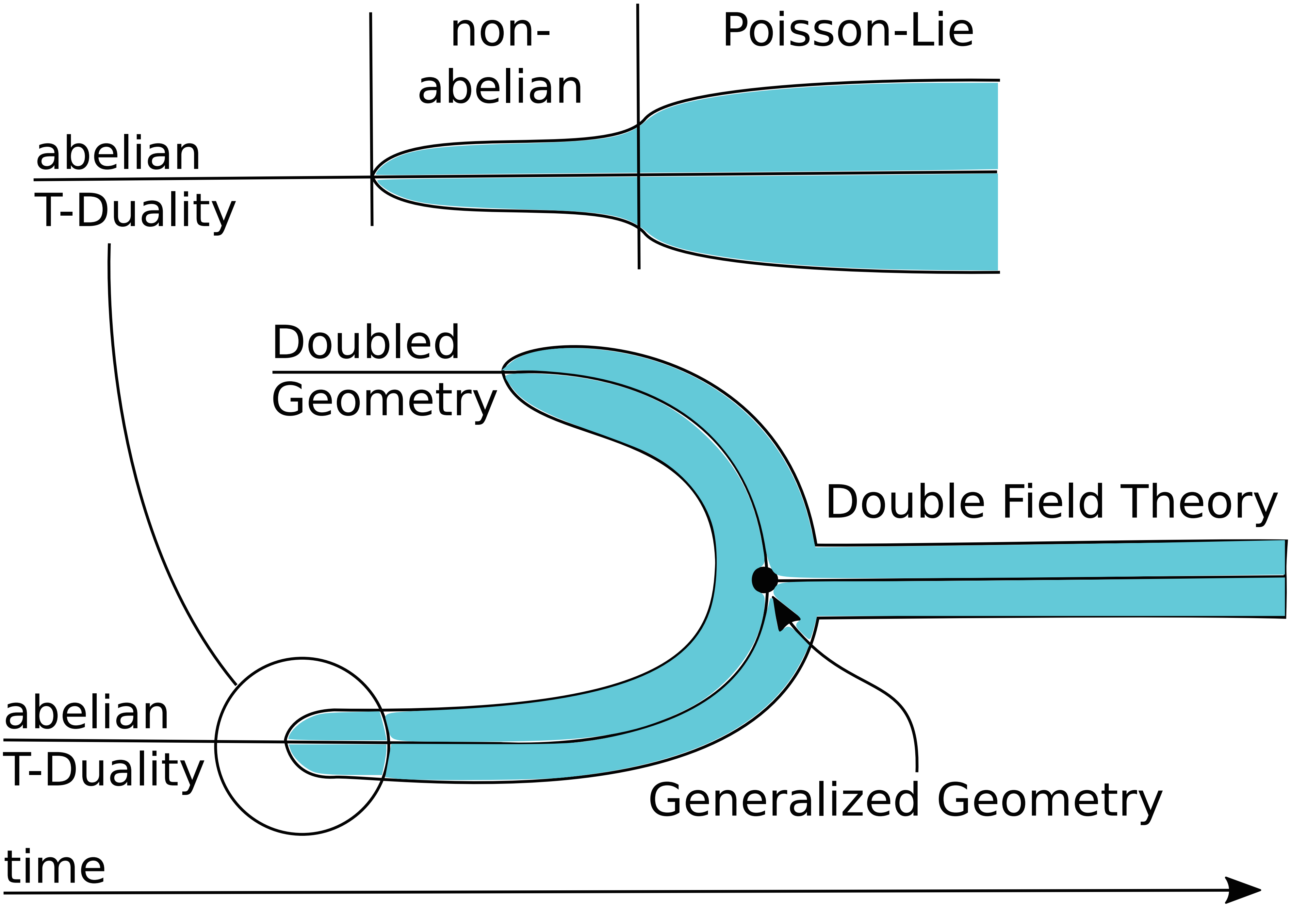

What do we gain?

- a target space description with manifest Poisson-Lie symmetry
- captures the dilaton
- captures the R/R sector
  - first derivation of R/R sector transformation for full Poisson-Lie T-duality
  - before only for abelian and non-abelian T-duality known
- modified SUGRA automatically build in
- simplified handling of integrable deformations
- consistent truncations in SUGRA

Outline

1. Motivation

2. Poisson-Lie T-duality

3. Double Field Theory on group manifolds

4. Summary

Drinfeld double

Definition: A **Drinfeld double** is a $2D$-dimensional Lie group $\mathcal{D}$, whose Lie-algebra $\mathfrak{d}$

1. has an ad-invariant bilinear for $\langle\cdot\,,\,\cdot\rangle$ with signature (D, D)
2. admits the decomposition into two maximal isotropic subalgebras $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$

- $\begin{pmatrix} t^a & t_a \end{pmatrix} = T_A \in \mathfrak{d}\,, \quad t_a \in \mathfrak{g}$ and $t^a \in \tilde{\mathfrak{g}}$
- $\langle T_A, T_B\rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix}$
- $[T_A, T_B] = F_{AB}{}^C T_C$ with non-vanishing commutators

$$[t_a, t_b] = f_{ab}{}^c t_c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$
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$$[t_a, t_b] = f_{ab}{}^c t_c + f'_{abc} t^c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$

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Poisson-Lie T-duality: 1. Definition

- 2D σ-model on target space M with action

  $S(E, M) = \int dz d\bar{z}\, E_{ij} \partial x^i \bar{\partial} x^j$
- $E_{ij} = g_{ij} + B_{ij}$ captures metric and two-from field on M
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- adjoint action of $g \in G$ on $t_A \in \mathfrak{d}$: $\mathrm{Ad}_g\, t_A = g t_A g^{-1} = M_A{}^B t_B$
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Definition: $S(E, \mathcal{D}/\tilde{G})$ and $S(\tilde{E}, \mathcal{D}/G)$ are **Poisson-Lie T-dual** if

$$E^{ij} = v_c{}^i M_a{}^c (M^{ae} M^b{}_e + E_0^{ab}) M_b{}^d v_d{}^j$$
$$\tilde{E}^{ij} = \tilde{v}^{ci} \tilde{M}^a{}_c (\tilde{M}_{ae} \tilde{M}_b{}^e + E_{0\,ab}) \tilde{M}^b{}_d \tilde{v}^{dj}$$

holds, where E_0^{ab} is constant and invertible with the inverse $E_{0\,ab}$.

Remark: The $\mathcal{E}$-model looks much nicer

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- we now know what η^{AB} and $F^{AB}{}_C$ is
- $\mathcal{E} : \mathfrak{d} \to \mathfrak{d}$ is captured by the *generalized metric*

$$\mathcal{H}_{AB} = \langle T_A, \mathcal{E} T_B\rangle = \begin{pmatrix} G^{ab} & G^{ac}B_{cb} \\ -B_{ac}G^{cb} & G_{ab} + B_{ac}G^{cd}G_{db}\end{pmatrix}$$

- with $\boxed{G_{ab} + B_{ab} = E_{0\,ab}}$

Poisson-Lie T-duality: 2. Properties

- captures $\begin{cases} \text{abelian T-d.} & G \quad \text{abelian} & \text{and} \quad \tilde{G} \quad \text{abelian} \\ \text{non-abelian T-d.} & G \quad \text{non-abelian} & \text{and} \quad \tilde{G} \quad \text{abelian} \end{cases}$

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- dilaton transformation

$$\phi = -\tfrac{1}{2} \log \left| \det \left(1 + \tilde{g}_0^{-1} (\tilde{B}_0 + \Pi) \right) \right|$$

$$\tilde{\phi} = -\tfrac{1}{2} \log \left| \det \left(1 + g_0^{-1} (B_0 + \tilde{\Pi}) \right) \right|$$

details tomorrow

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2D σ-model perspective

(modified) SUGRA perspective

Additional structure on the Drinfeld double

- *right* invariant vector $E_A{}^I$ field on $\mathcal{D}$ is the inverse transposed of *left* invariant Maurer-Cartan form $t_A E^A{}_I dX^I = g^{-1} dg$

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- two η-compatible, covariant derivatives[1]
  1. flat derivative

     $$D_A V^B = E_A{}^I \partial_I V^B$$

  2. convenient derivative

     $$\nabla_A V^B = D_A V^B + \tfrac{1}{3} F_{AC}{}^B V^C - w F_A V^B , \qquad F_A = D_A \log \left| \det(E^B{}_I) \right|$$

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- generalized metric $\mathcal{H}_{AB}$ ($w = 0$)
  $$\mathcal{H}_{AB} = \mathcal{H}_{(AB)}, \qquad \mathcal{H}_{AC}\eta^{CD}\mathcal{H}_{DB} = \eta_{AB}$$
- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$

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- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$
- triple $(\mathcal{D}, \mathcal{H}_{AB}, d)$ captures the doubled space of DFT

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Double Field Theory for $(\mathcal{D}, \mathcal{H}_{AB}, d)$

- action ($\nabla_A d = -\frac{1}{2} e^{2d} \nabla_A e^{-2d}$)

$$S_{\mathrm{NS}} = \int_{\mathcal{D}} d^{2D}X e^{-2d} \Big(\frac{1}{8} \mathcal{H}^{CD} \nabla_C \mathcal{H}_{AB} \nabla_D \mathcal{H}^{AB} - \frac{1}{2} \mathcal{H}^{AB} \nabla_B \mathcal{H}^{CD} \nabla_D \mathcal{H}_{AC} - 2 \nabla_A d \nabla_B \mathcal{H}^{AB} + 4 \mathcal{H}^{AB} \nabla_A d \nabla_B d + \frac{1}{6} F_{ACD} F_B{}^{CD} \mathcal{H}^{AB} \Big)$$

- $2D$-diffeomorphisms

$$L_\xi V^A = \xi^B D_B V^A + w D_B \xi^B V^A$$

- global O(D,D) transformations

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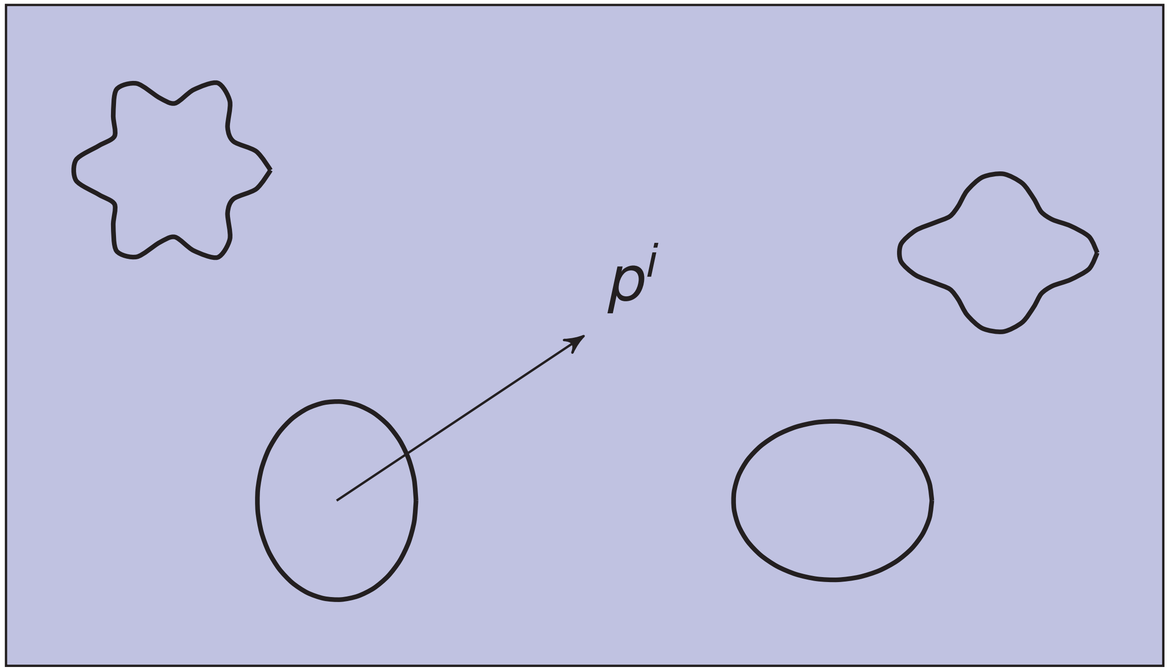
- section condition (SC)

$$\eta^{AB} D_A \cdot D_B \cdot = 0$$

How we got this action?

- closed strings in D-dim. flat space
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\mathrm{NS}} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4\partial_i\phi\partial^i\phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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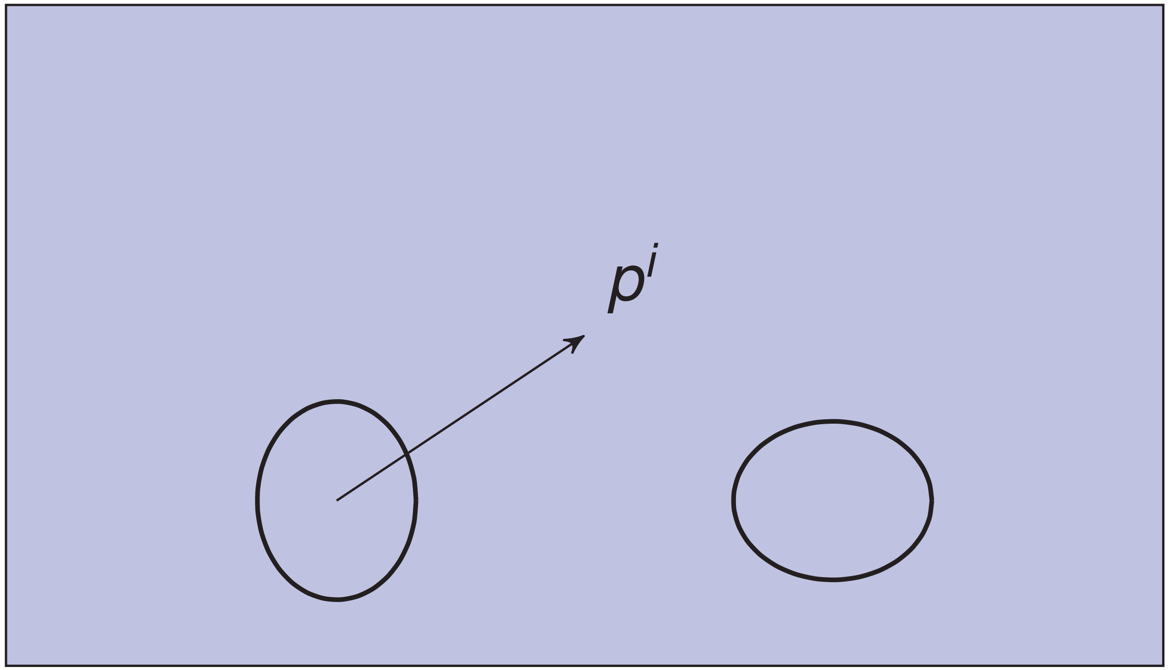

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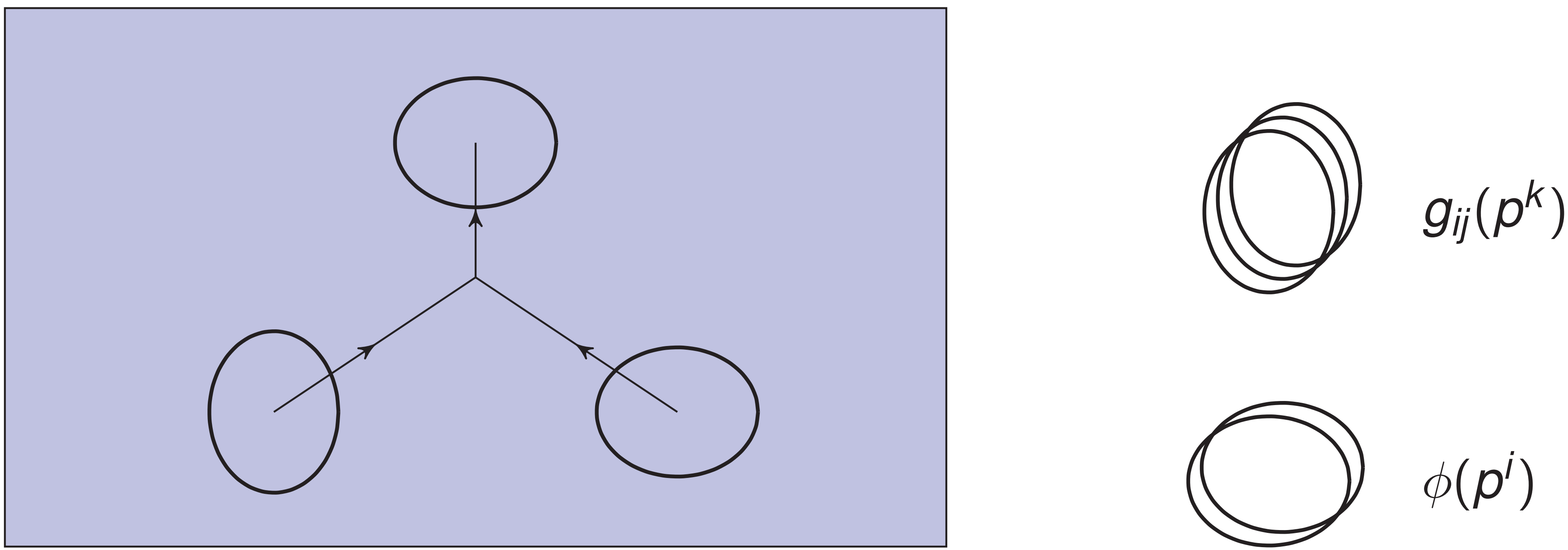
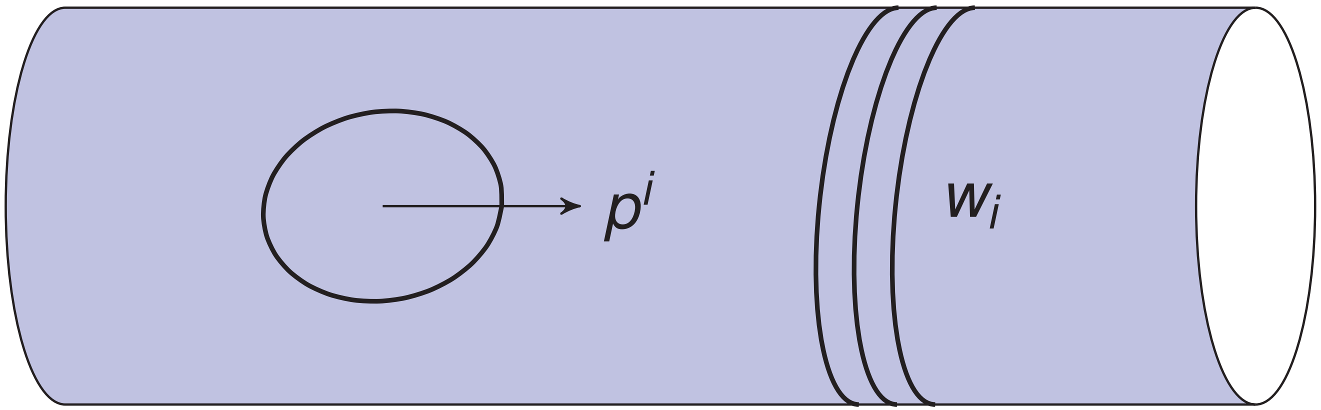


Double Field Theory

- closed strings on a flat torus
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = (\tilde{x}_i \quad x^i)$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D}X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



DFT on group manifolds = DFT$_{\text{WZW}}$

Use group manifold (Wess-Zumino-Witten model) instead of a torus to derive DFT!

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TASKS
- Derive cubic action and gauge transformations (CSFT)
- Rewrite in terms of η_{AB}, F_{ABC} and $\mathcal{H}_{AB}$
- Figure out that $\mathcal{D}$ does not have to be $G_{\text{L}} \times G_{\text{R}}$

} not trivial :-)

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Symmetries of the action

- S_{NS} invariant for $X^I \to X^I + \xi^A E_A{}^I$ and
  1. $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$ and $e^{-2d} \to e^{-2d} + \mathcal{L}_\xi e^{-2d}$
  2. $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + L_\xi \mathcal{H}^{AB}$ and $e^{-2d} \to e^{-2d} + L_\xi e^{-2d}$

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object	gen.-diffeomorphisms	$2D$-diffeomorphisms	global $\mathrm{O}(D,D)$
$\mathcal{H}_{AB}$	tensor	scalar	tensor
$\nabla_A d$	not covariant	scalar	1-form
e^{-2d}	scalar density (w=1)	scalar density (w=1)	invariant
η_{AB}	invariant	invariant	invariant
$F_{AB}{}^C$	invariant	invariant	tensor
$E_A{}^I$	invariant	vector	1-form
S_{NS}	invariant	invariant	invariant
SC	invariant	invariant	invariant
D_A	not covariant	covariant	covariant
∇_A	not covariant	covariant	covariant

manifest (2D-diffeomorphisms and global O(D,D))

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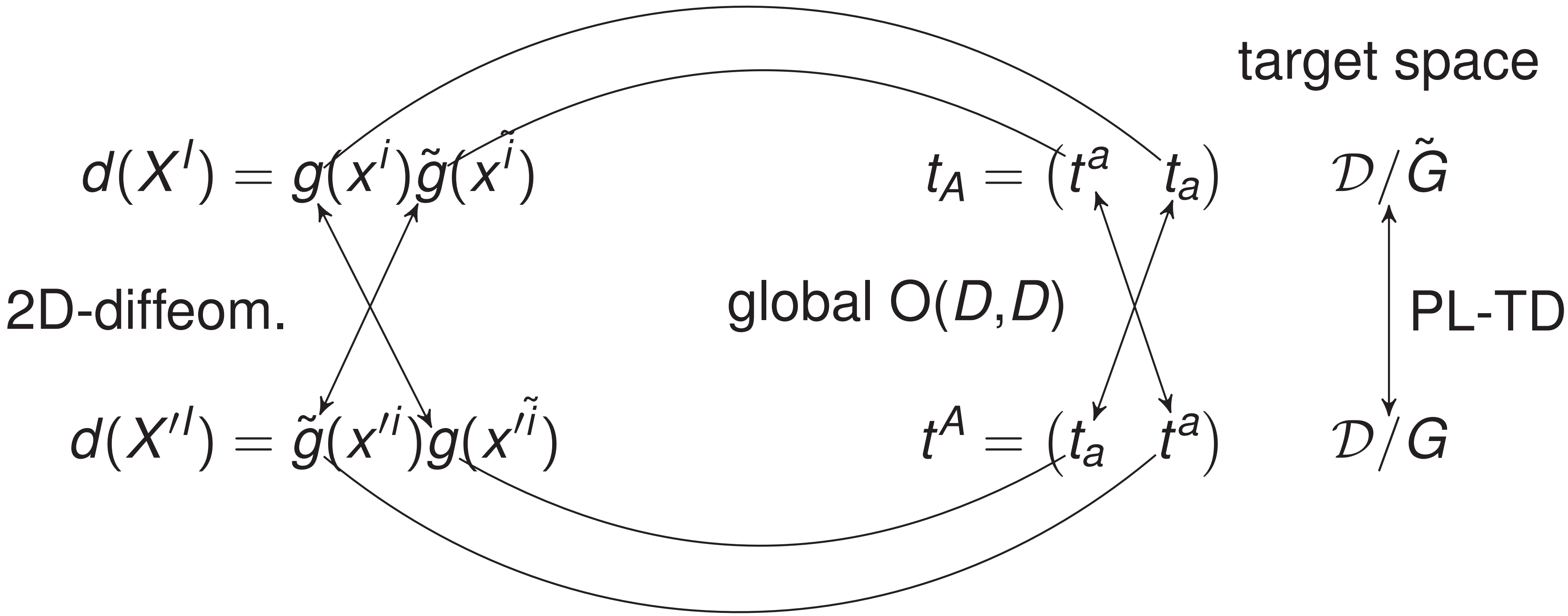

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Equivalence to supergravity: 1. Generalized parallelizable spaces

- generalized tangent space element $V^{\hat{I}} = \begin{pmatrix} V^i & V_i \end{pmatrix}$
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$$\widehat{\mathcal{L}}_\xi V^{\hat{I}} = \xi^{\hat{J}} \partial_{\hat{J}} V^{\hat{I}} + \left(\partial^{\hat{I}} \xi_{\hat{J}} - \partial_{\hat{J}} \xi^{\hat{I}}\right) V^{\hat{J}} \qquad \text{with} \qquad \partial_{\hat{I}} = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$$

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Definition: A manifold M which admits a globally defined generalized frame field $\widehat{E}_A{}^{\hat{I}}(x^i)$ satisfying

1. $\widehat{\mathcal{L}}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} = F_{AB}{}^C \widehat{E}_C{}^{\hat{I}}$

   where $F_{AB}{}^C$ are the structure constants of a Lie algebra $\mathfrak{h}$
2. $\widehat{E}_A{}^{\hat{I}} \eta^{AB} \widehat{E}_B{}^{\hat{J}} = \eta^{\hat{I}\hat{J}} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^i_j & 0 \end{pmatrix}$

is a **generalized parallelizable space** $(M, \mathfrak{h}, \widehat{E}_A{}^{\hat{I}})$.

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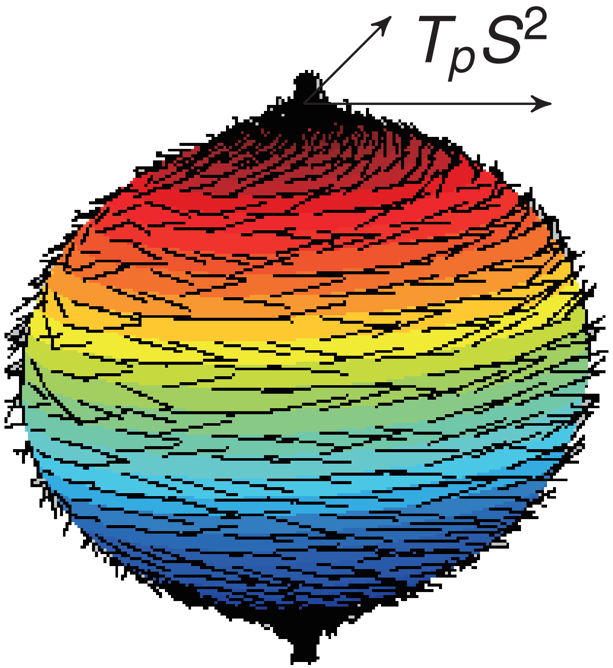

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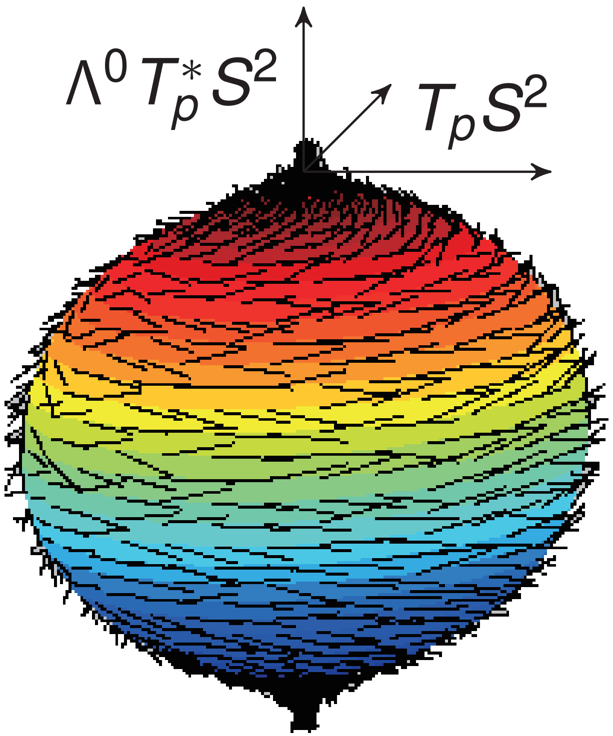

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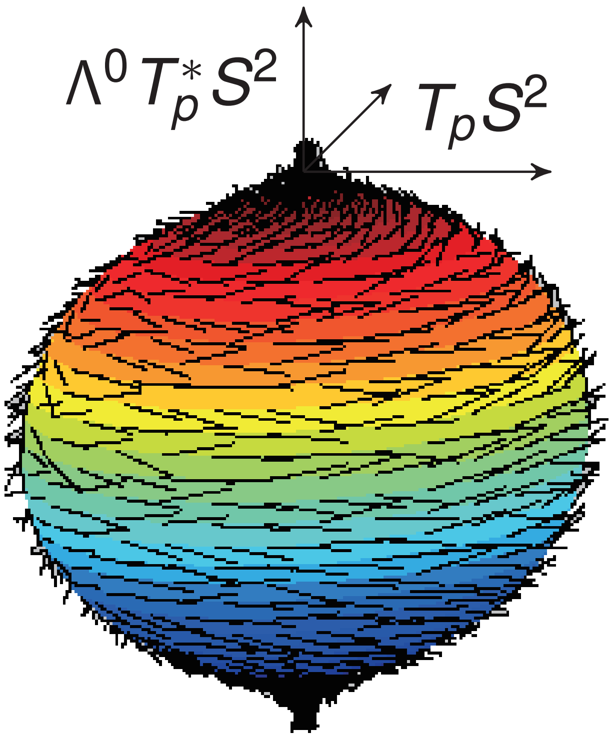

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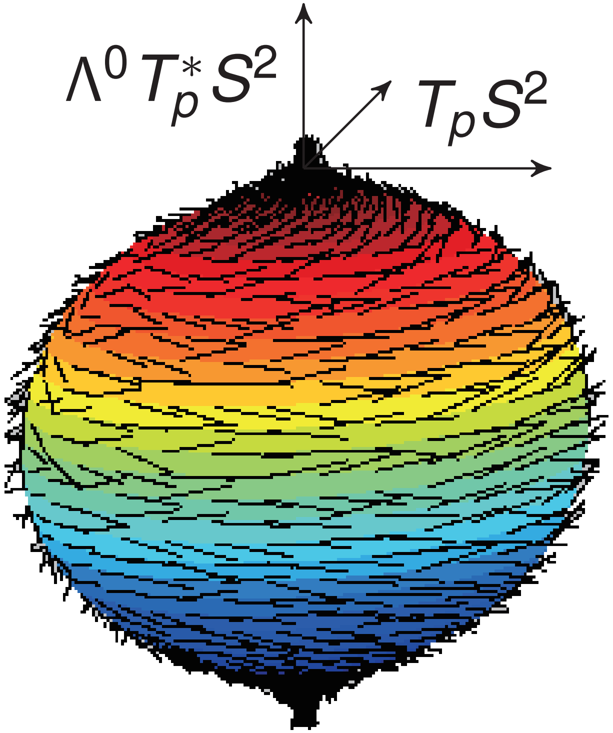

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¿ Is there a systematic way to construct them ?

Equivalence to supergravity: 2. Generalized metric and dilaton

- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\hat{I}}) \qquad \text{and} \qquad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\hat{I}})$$

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$$\mathcal{H}^{AB} = \widehat{E}^A{}_{\hat{I}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \widehat{E}^B{}_{\hat{J}} \qquad \text{with} \qquad \widehat{\mathcal{H}}^{\hat{I}\hat{J}} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lk} & -B_{ik} g^{kl} \\ g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

- express d in terms of the standard generalized dilaton $\widehat{d}$

$$d = \widehat{d} - \frac{1}{2} \log |\det \tilde{v}_{ai}|$$

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- plug into the DFT action S_{NS}

Equivalence to supergravity: 3. IIA/B bosonic sector action

- if G and $\tilde{G}$ are unimodular

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int d^D x \, e^{-2\widehat{d}} \Big(\frac{1}{8} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{K}} \widehat{\mathcal{H}}_{\hat{I}\hat{J}} \partial_{\hat{L}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - 2 \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - \frac{1}{2} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{L}} \widehat{\mathcal{H}}_{\hat{I}\hat{K}} + 4 \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{d} \Big)$$

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- similar story for R/R sector (tomorrow)

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie T-duality

- in general $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality } (2D\text{-diff.})} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
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Summary

- DFT, Poisson-Lie T-duality and Drinfeld doubles fit together naturally
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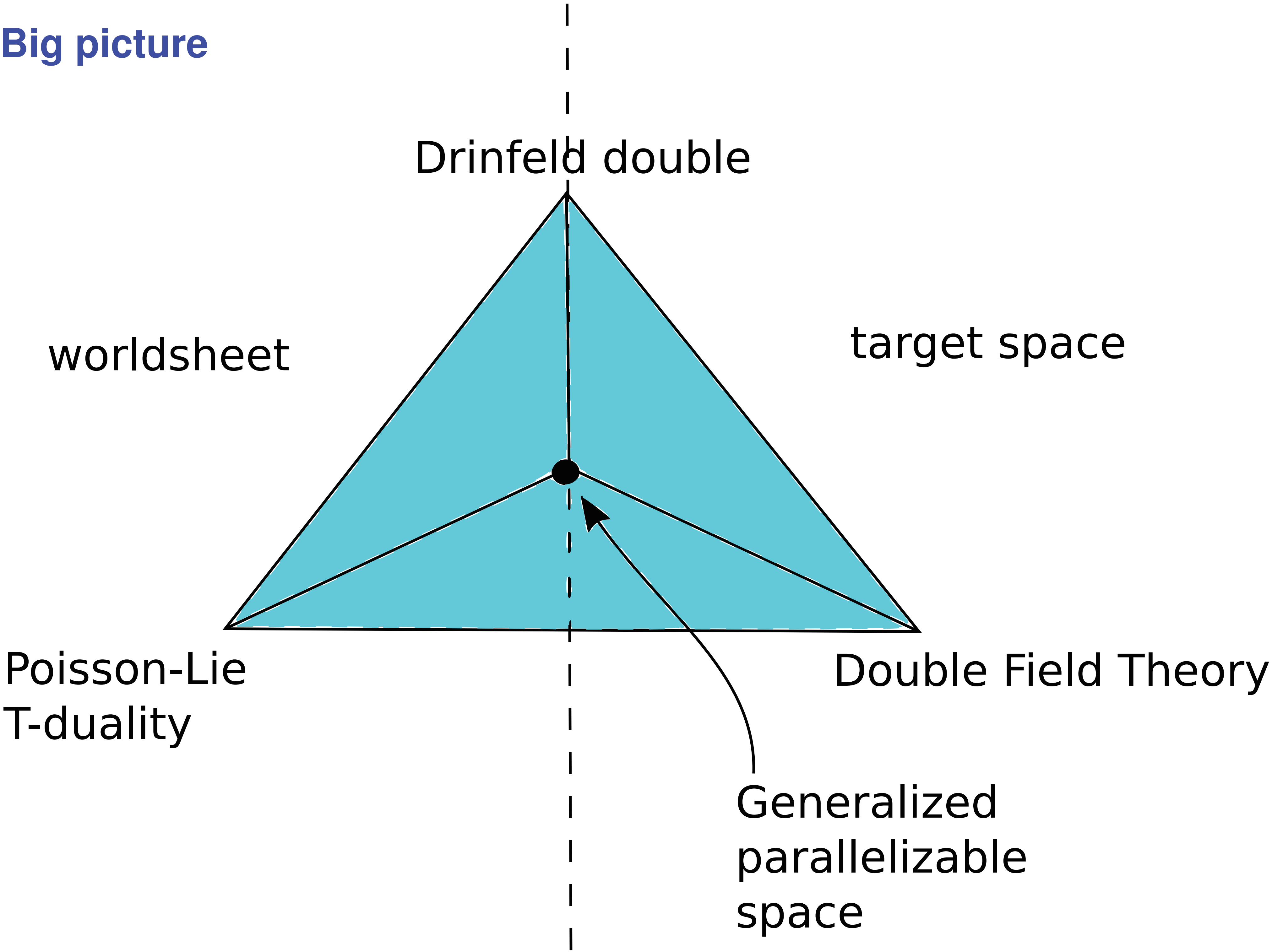
- plan for tomorrow
  - dilaton transformation
  - R/R sector transformation
  - modified SUGRA
  - integrable deformations
  - dressing coset construction

Big picture



Drinfeld double
worldsheet
target space
Poisson-Lie
T-duality
Double Field Theory
Generalized
parallelizable
space

Poisson-Lie Symmetry and Double Field Theory

Part II

Falk Hassler

University of Oviedo

based on

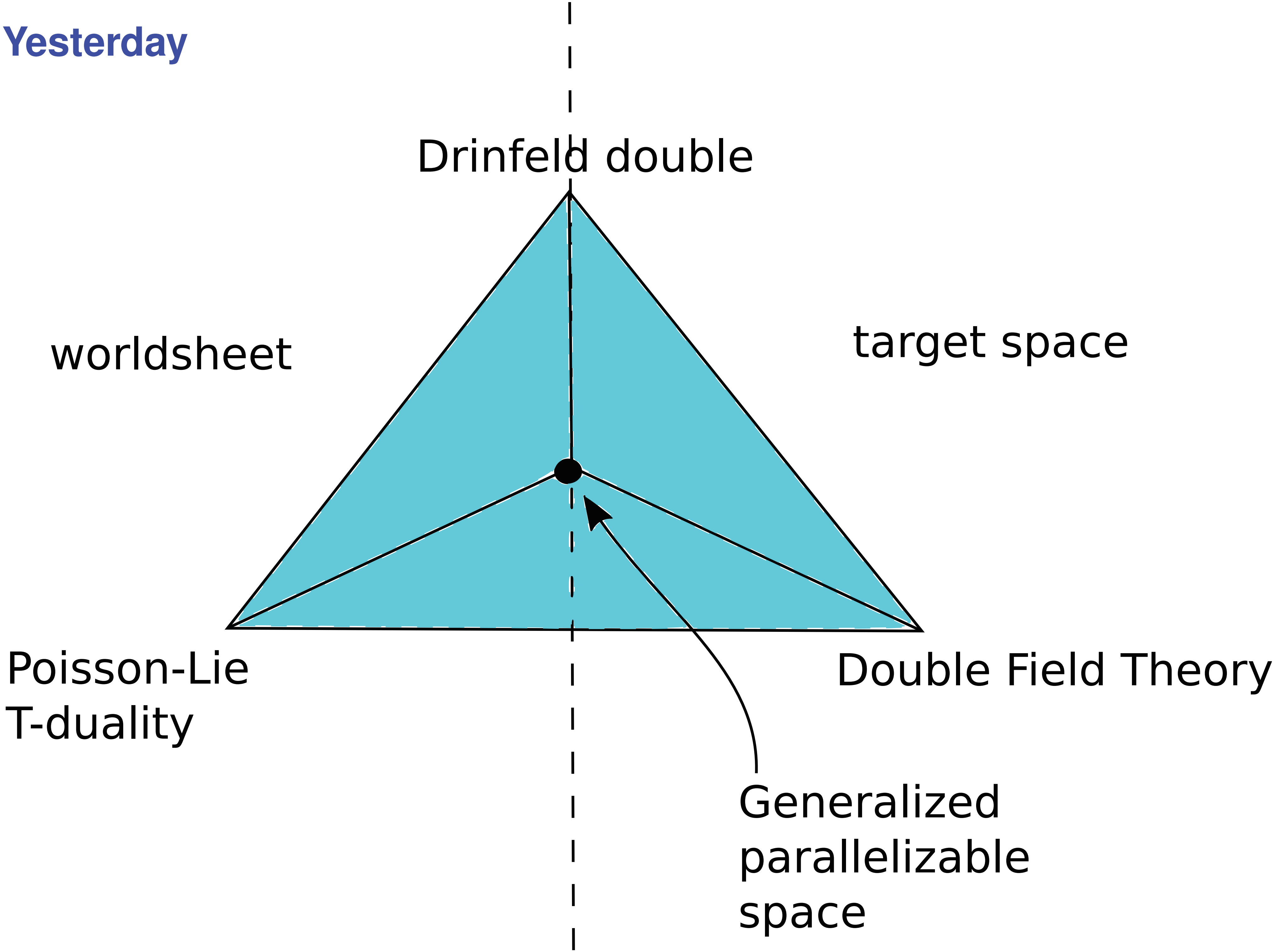
1810.11446,
1707.08624, 1611.07978,
1502.02428, 1410.6374

and work in progress

March 7th, 2019

Universidad de Oviedo
Universidá d'Uviéu
University of Oviedo

Yesterday



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Ingredients for NS/NS sector of DFT on group manifolds

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- generalized frame field makes contact with SUGRA fields

Outline

1. Quick reminder

2. Dilaton transformation

3. R/R sector of Double Field Theory on $\mathcal{D}$

4. Application to integrable deformations

5. Outlook

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie T-duality

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Remarks:

- $F_A = D_A \log \left| \det(E^B{}_I) \right|$
- biggest possible isometry group $\mathcal{D}_\mathrm{L} \times \mathcal{D}_\mathrm{R}$
- for Poisson-Lie T-duality just $\mathcal{D}_\mathrm{L}$ required
- if additionally $\mathcal{F} \subset \mathcal{D}_\mathrm{R}$ gauge it $\rightarrow$ dressing coset

Dilaton transformation

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- $g = v^T e^T e v$ with $\begin{cases} (\tilde{B}_0 + \tilde{g}_0)^{ab} = E^{0\,ab} \\ \Pi^{ab} = M^{ac} M^b{}_c \\ e^{-1}e^{-T} = \tilde{g}_0 - (\tilde{B}_0 + \Pi)\tilde{g}_0^{-1}(\tilde{B}_0 + \Pi) \\ \tilde{e}_0^T \tilde{e}_0 = \tilde{g}_0 \\ e^{-T} = \tilde{e}_0 + \tilde{e}_0^{-T}(\tilde{B}_0 + \Pi) \end{cases}$

Dilaton transformation

- $(D_A - F_A)e^{-2d} = 0 \quad \to \quad \partial_I(\underbrace{2d + \log|\det v| + \log|\det \tilde{v}|}_{= 2\phi_0 = \text{const.}}) = 0$
- $d = \phi - 1/4 \log|\det g| - \frac{1}{2}\log|\det \tilde{v}|$
  $\phi = \phi_0 + \frac{1}{4}\log|\det g| - \frac{1}{2}\log|\det v|$
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- $\phi = \phi_0 + \frac{1}{2}\log|\det e| = \phi_0 - \frac{1}{2}\log|\det \tilde{e}_0| - \frac{1}{2}\log\left|\det\left(1 + \tilde{g}_0^{-1}(\tilde{B}_0 + \Pi)\right)\right|$
- reproduces [Jurco and Vysoky, 2018]

O(D,D) Majorana-Weyl spinor on $\mathcal{D}$

- Γ-matrices: $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$
- chirality Γ_{2D+1} with $\{\Gamma_{2D+1}, \Gamma_A\} = 0$
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- Γ^a = creation op. and Γ_a = annihilation op. ($\{\Gamma^a, \Gamma_b\} = 2\delta^a_b$)
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- O(D,D) transformation in spinor representation

  $\mathcal{S}_{\mathcal{O}}\Gamma_A\mathcal{S}_{\mathcal{O}}^{-1} = \Gamma_B\mathcal{O}^B{}_A \qquad \mathcal{O}^T\eta\mathcal{O} = \eta$

R/R sector of DFT on group manifolds

- action $S_{\mathrm{RR}} = \frac{1}{4} \int d^{2d}X \, (\nabla\!\!\!/ \chi)^\dagger \, S_{\mathcal{H}} \, \nabla\!\!\!/ \chi$
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- $\not\nabla^2 = 0$ under SC
- χ is chiral (IIB) or anti-chiral (IIA)
- satisfies self duality condition

  $G = -\mathcal{K} G \quad \text{with} \quad G = \not\nabla \chi \quad \text{and} \quad \mathcal{K} = C^{-1} S_{\mathcal{H}}$

Symmetries of the action

- $S_{\mathrm{R/R}}$ invariant for $X^I \to X^I + \xi^A E_A{}^I$ and
  1. $\chi \to \chi + \mathcal{L}_\xi \chi$ and $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$
  2. $\chi \to \chi + L_\xi \chi$ and $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + L_\xi \mathcal{H}^{AB}$

1. generalized diffeomorphisms

$$\mathcal{L}_\xi \chi = \xi^A \nabla_A \chi + \frac{1}{2} \nabla_A \xi_B \Gamma^{AB} \chi + \frac{1}{2} \nabla_A \xi^A \chi$$

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2. $2D$-diffeomorphisms

$$L_\xi \chi = \xi^A D_A \chi - \tfrac{1}{2} (\xi^A F_A - D_A \xi^A) \chi \quad \text{and} \quad L_\xi \mathcal{H}^{AB} = \xi^C D_C \mathcal{H}^{AB}$$

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3. global O(D,D) transformations ($\mathcal{O}^A{}_C \mathcal{O}^B{}_D \eta^{CD} = \eta^{AB}$)

$$\chi \to S_\mathcal{O} \chi \quad \text{and} \quad \mathcal{H}^{AB} \to \mathcal{O}^A{}_C \mathcal{H}^{CD} \mathcal{O}^B{}_D$$

- section condition (SC) for f_1, f_2 with weights w_1, w_2

$$(D_A f_1 - w_1 F_A f_1)(D^A f_2 - w_2 F^A f_2) = 0$$

Equivalence to (m)SUGRA: 1. R/R field strengths

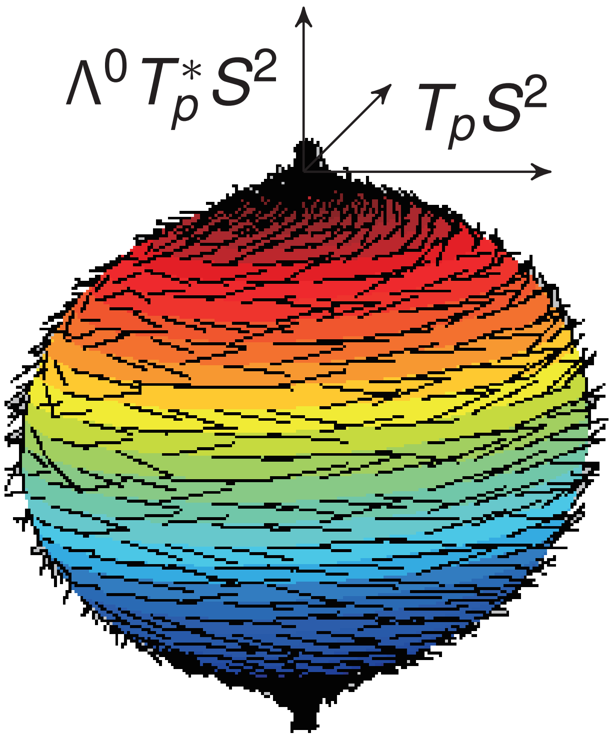
- transport χ to the generalized tangent space:

  $$\widehat{\chi} = |\det \tilde{e}_{ai}|^{-1/2} S_{\widehat{E}} \chi \qquad (t^a \tilde{e}_{ai} = \tilde{g}^{-1} d\tilde{g})$$

- remember generalized metric from yesterday:

  $$\widehat{\mathcal{H}}^{\hat{I}\hat{J}} = \widehat{E}_A{}^{\hat{I}} \mathcal{H}^{AB} \widehat{E}_B{}^{\hat{J}}$$

Remember S^2 is not parallelizable, but generalized parallelizable



Def.: M is parallelizable if $\exists$ d=dim M smooth vector fields providing a basis e_a for T_pM at every point p on M.

- examples: S^3, S^7, Lie groups
- Scherk-Schwarz compactifications on M do not break any SUSY
- counterexample S^2 (hairy ball)

use generalized tangent space instead of TM

- all spheres are generalized parallelizable on $TM \oplus \Lambda^{d-2} T^* M$
- generalized frame field $\hat{E}_A$ fulfilling $\widehat{\mathcal{L}}_{\hat{E}_A} \hat{E}_B = F_{AB}{}^C \hat{E}_C$
- consistent ansätze from compactification with max. SUSY

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$$|\det \tilde{e}_{ai}|^{-1/2} S_{\widehat{E}} \nabla\!\!\!/ \chi = \left(\partial\!\!\!/ - \mathbf{X}_{\hat{I}} \widehat{\Gamma}^{\hat{I}} \right) \widehat{\chi} \quad \text{with} \quad \mathbf{X}_{\hat{I}} = \begin{pmatrix} I^i \\ -V_i \end{pmatrix}$$

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- $\mathbf{X}_{\hat{I}}$ vanishes if $\tilde{\mathfrak{g}}$ is unimodular
- introduce field strength $\widehat{F} = e^{\phi} S_B \left(\partial\!\!\!/ - \mathbf{X}_{\hat{I}} \widehat{\Gamma}^{\hat{I}} \right) \widehat{\chi}$
- and derivative $\mathbf{d} = e^{\phi} S_B \left(\partial\!\!\!/ - \mathbf{X}_{\hat{I}} \widehat{\Gamma}^{\hat{I}} \right) S_B^{-1} e^{-\phi}$

Equivalence to (m)SUGRA: 2. field equations & Bianchi identity

- DFT R/R field equations: $\not\nabla(\mathcal{K}G) = 0$ remember $G = \not\nabla\chi$
- rewrite them as:

$$\mathbf{d} \star \widehat{F} = 0 \qquad \star = C^{-1} S_g^{-1}$$

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- action on polyforms

  $\mathbf{d} \quad \leftrightarrow \quad d + H \wedge -Z \wedge -\iota_I \quad \text{with} \quad Z = d\phi + \iota_I B - V$

  $\star \quad \leftrightarrow \quad \star$

- matches the R/R sector of (m)SUGRA
- some holds for the NS/NS sector

Restrictions on $\mathcal{H}_{AB}$ and χ to admit Poisson-Lie Symmetry

- remember $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality } (2D\text{-diff.})} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
- $x'^{\tilde{i}}$ part not compatible with ansatz for SC solutions $\to$ avoid it

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A doubled space $(\mathcal{D}, \mathcal{H}_{AB}, d)$ has Poisson-Lie symmetry iff

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- $\not\nabla \chi = 0$ for Poisson-Lie symmetric χ is algebraic

  $\not\nabla \chi = \frac{1}{12} F_{ABC} \Gamma^{ABC} \chi$
- finding R/R solutions reduces to linear algebra
- similar for NS/NS sector

  (here field equations are in general quadratic)

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- starting point is solution to (m)CYBE

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1. $c^2 = -1$ Yang-Baxter σ-model or η-deformation
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- generalized metric after global O(D,D) very simple

$$\mathcal{H}^{AB} = \begin{pmatrix} k_{ab} & 0 \\ 0 & k^{ab} \end{pmatrix}$$

- structure coefficients have non-trivial components

$$F_{abc} = 0\,, \quad F_{ab}{}^{c} = \kappa^{-1/2} f_{ab}{}^{c}\,,$$
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- field equations for NS/NS + R/R sector become linear

Field equations: 1. Variation of the NS/NS action

- two contributions

  1. $\delta S_{\mathrm{NS}} = -2 \int d^{2D} X e^{-2d} \mathcal{R} \delta d$
  2. $\delta S_{\mathrm{NS}} = \int d^{2D} X e^{-2d} \mathcal{K}_{AB} \delta \mathcal{H}^{AB}$

$$\mathcal{R} = 4\mathcal{H}^{AB}\nabla_A\nabla_B d - \nabla_A\nabla_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\nabla_A d\,\nabla_B d + 4\nabla_A d\,\nabla_B\mathcal{H}^{AB}$$
$$+ \frac{1}{8}\mathcal{H}^{CD}\nabla_C\mathcal{H}_{AB}\nabla_D\mathcal{H}^{AB} - \frac{1}{2}\mathcal{H}^{AB}\nabla_B\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AC} + \frac{1}{6}F_{ACD}F_B{}^{CD}\mathcal{H}^{AB}$$

$$\mathcal{K}_{AB} = \frac{1}{8}\nabla_A\mathcal{H}_{CD}\nabla_B\mathcal{H}^{CD} - \frac{1}{4}\left[\nabla_C - 2(\nabla_C d)\right]\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AB} + 2\nabla_{(A}\nabla_{B)}d$$
$$- \nabla_{(A}\mathcal{H}^{CD}\nabla_D\mathcal{H}_{B)C} + \left[\nabla_D - 2(\nabla_D d)\right]\left[\mathcal{H}^{CD}\nabla_{(A}\mathcal{H}_{B)C} + \mathcal{H}^C{}_{(A}\nabla_C\mathcal{H}^D{}_{B)}\right]$$
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$$\mathcal{R} = 4\mathcal{H}^{AB}\nabla_A\nabla_B d - \nabla_A\nabla_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\nabla_A d\,\nabla_B d + 4\nabla_A d\,\nabla_B\mathcal{H}^{AB}$$
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$$+ \frac{1}{6}F_{ACD}F_B{}^{CD}$$

- $\mathcal{H}_{AB}$ not just symmetric but restricted to $O(D,D)$ $\rightarrow$ project $\mathcal{K}_{AB}$

Field equations: 2. Poisson-Lie symmetry

- generalized Ricci curvature

$$\mathcal{R}_{AB} = 2P_{(A}{}^{C}\mathcal{K}_{CD}\overline{P}_{B)}{}^{D}$$

$$P_{AB} = \tfrac{1}{2}(\eta_{AB} + \mathcal{H}_{AB}) \quad \text{and} \quad \overline{P}_{AB} = \tfrac{1}{2}(\eta_{AB} - \mathcal{H}_{AB})$$

- finally the field equations are:

$$\mathcal{R} = 0$$

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- Poisson-Lie symmetry simplifies $\mathcal{R}$ and $\mathcal{R}_{AB}$

$$\mathcal{R} = \frac{1}{12}F_{ACE}F_{BDF}\left(3\mathcal{H}^{AB}\eta^{CD}\eta^{EF} - \mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\right)$$

$$\mathcal{R}_{AB} = \frac{1}{8}(\mathcal{H}_{AC}\mathcal{H}_{BF} - \eta_{AC}\eta_{BF})(\mathcal{H}^{KD}\mathcal{H}^{HE} - \eta^{KD}\eta^{HE})F_{KH}{}^{C}F_{DE}{}^{F}$$

Generalized frame field and target space fields

- generalized frame field: $\widehat{E}_A{}^{\hat{I}} = \begin{pmatrix} \kappa^{1/2} e^a{}_i & \kappa^{-1/2}(\Pi^{ab} + R^{ab}) e_b{}^i \\ 0 & \kappa^{-1/2} e_a{}^i \end{pmatrix}$

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- metric G and B-field from generalized metric $\widehat{H}^{\hat{I}\hat{J}}$

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- modified SUGRA vector: $\mathbf{X}^{\hat{I}} = \frac{1}{2}\begin{pmatrix} R^{bc} f_{bc}{}^a v_a{}^i \\ 0 \end{pmatrix}$

Generalized frame field and target space fields

- generalized frame field: $\widehat{E}_A{}^{\hat{I}} = \begin{pmatrix} \kappa^{1/2} e^a{}_i & \kappa^{-1/2}(\Pi^{ab} + R^{ab}) e_b{}^i \\ 0 & \kappa^{-1/2} e_a{}^i \end{pmatrix}$
- metric G and B-field from generalized metric $\widehat{H}^{\hat{I}\hat{J}}$

$$g + B = e^T \left((\kappa k)^{-1} + R + \Pi \right) e \qquad t_a e^a{}_i dx^i = g^{-1} \mathrm{d} g$$

- dilaton $\phi = \phi_0 + \frac{1}{4} \log|\det g| + \frac{1}{2} \log|\det e|$
- modified SUGRA vector: $\mathbf{X}^{\hat{I}} = \frac{1}{2} \begin{pmatrix} R^{bc} f_{bc}{}^a v_a{}^i \\ 0 \end{pmatrix}$
- R/R fields:

$$\widehat{G}^{(1)} = -\frac{1+\kappa^2}{\sqrt{2}} (\Pi + R)^{ab} f_{abc} e^c$$

$$\widehat{G}^{(3)} = \frac{1+\kappa^2}{3\sqrt{2}} f_{abc} e^a \wedge e^b \wedge e^c$$

There are many interesting questions

- translation of all the intriguing results in Poisson-Lie T-duality e.g.
  - implement dressing cosets
  - study global properties (non-abelian momentum and winding exchange)
  - D-branes
- better understand supersymmetry
- apply to background with just partial PL-symmetry
- quantization of $\mathcal{E}$-model $\leftrightarrow$ α' corrections
- EFT has similar structure as DFT.
  Can we formulate "Poisson-Lie" U-duality?


PL-TD & DFT