Generalized Parallelizable Spaces, Consistent Truncations & Dualities   1705.09304, 1707.08624

1. Introduction

parallelizable M: $\exists \dim M$ smooth vector fields $\{E_1, \dots, E_d\}$ on M provide basis for $T_p M$ for all $p \in M$

- examples: S^1, S^3, S^7, Lie groups
- counter example: S^2 (hairy ball theorem)
- trivial TM $\Rightarrow$ * compactifications preserve all SUSY
  * — " — are consistent

$$\begin{tikzcd}[column sep=huge]
\text{11 / 10D SUGRA action} \arrow[r, "\text{comp. ansatz}"] \arrow[d, "\text{solve eom (hard)}"'] & \text{(11/10-d)D g SUGRA action} \arrow[d, "\text{(easier)}"] \\
\text{Vacuum} & \text{Vacuum} \arrow[l, "\text{uplift}"]
\end{tikzcd}$$

BUT there are more like $AdS_7 \times S_4$, $AdS_5 \times S_5$ (hep-th/9903214)
all have FLUXES $\rightarrow$ generalized parallelizable M:
vector fields provide basis for gen. tangent bundle

e.g. $TM \oplus \Lambda^{n-2} T^*M \ni V, W$

$$V = v + \lambda \qquad W = w + \mu$$

gen. Lie derivative

$$\mathcal{L}_V W = [v, w] + L_v \mu - \iota_w d\lambda$$

$$\boxed{\mathcal{L}_{\hat{E}_A} \hat{E}_B = X_{AB}{}^{C} \hat{E}_C} \qquad X_{AB}{}^{C} \text{ constant}$$

- read of compactification ansatz from $\hat{E}_A$

so far: ① choose M
② chose structure const. $X_{AB}{}^C$
③ guess $\hat{E}_A$

→ all spheres are gen. parallelizable (1401.3360)

② Systematic approach

embedd M into higher dim. parallelizable space = Lie group G

with frame field $E_A{}^I$ &

flat derivative $D_A = E_A{}^I \partial_I$, $[D_A, D_B] = X_{AB}{}^C D_C$

gen. Lie derivative: $\mathcal{L}_\xi V^A = [\xi, V]^A + Y^{AB}{}_{CD} D_B \xi^C V^D$

implements structure of gen. tangent space

- inv tensor of $E_{d(d)} = U$

d	2	3	4	5	6	
U	$SL(2)\times\mathbb{R}^+$	$SL(3)\times SL(2)$	$SL(5)$	$Spin(5,5)$	$E_{6(6)}$	
$R1$	$2_1 + 1_{-1}$	$(3,2)$	10	16	27	...

$$TM \oplus \Lambda^2 T^*M \hat{=} 4 + 6 = 10$$

closure $[\mathcal{L}_{\xi_1}, \mathcal{L}_{\xi_2}] V = \mathcal{L}_{(\mathcal{L}_{\xi_1}\xi_2)} V$ requires

(I) restriction on $X_{AB}{}^C$

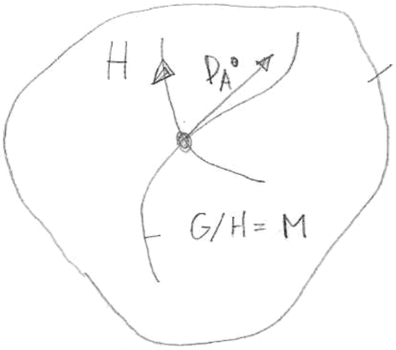
$R1 \times R1 \times \overline{R1} \to R_1 \times \mathrm{adj}(U) \to$ embedding tensor irreps

e.g. $d=4$ $\quad 10 \times 24 \to 10 + 15 + 40$

classifies all max. gSUGRAs in $(11-d)$ dim

② Section condition

$$Y^{AB}{}_{CD}\, D_A D_B \cdot = 0 \qquad Y^{AB}{}_{CD}\, D_A \cdot D_B \cdot = 0 \quad (1)$$



foliation, describe in terms of H-principal bundle over M

* connection $A = t_a A^a{}_i dx^i$

dim $H \cdot d$ components e.g.
$6 \cdot 4$ for $SL(5)$ only
4 are independent

Solution $\hat{=}$ flat connection $\qquad \mathfrak{g} = \mathfrak{m} \oplus \mathfrak{h}$

for a) $[\mathfrak{m},\mathfrak{m}] \subset \mathfrak{h} \qquad [\mathfrak{m},\mathfrak{h}] \subset \mathfrak{m} \qquad [\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}$

M symmetric space

b) $[\mathfrak{m},\mathfrak{h}] \subset \mathfrak{m} \qquad [\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}$

M Lie group with FLUXES proven

③ T-Duality

different choices for H with same G

example $O(d-1, d-1) \subset E_{d(d)}$ with $TM \oplus T^*M$

- H maximally isotropic subgroup of G

b) → Drinfeld double → Poisson Lie T-duality

includes abelian + non-ab. T-duality

different 10D vacua

(10−d) D vacuum